Insurance: Mathematics and Economics 80 (2018) 93–109

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal robust reinsurance-investment strategies for insurers with mean reversion and mispricing[☆]

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HIGHLIGHTS

- An optimal robust reinsurance-investment problem for insurers is studied.
- Mean reversion, mispricing and model ambiguity are incorporated in our model.
- Optimal reinsurance-investment strategy and optimal value function are derived.
- Dynamic programming approach is used and two special cases are discussed.
- Numerical examples are presented to illustrate the impact of some parameters.

ARTICLE INFO

Article history:
Received August 2017
Received in revised form January 2018
Accepted 12 March 2018
Available online 17 March 2018

Keywords:
Proportional reinsurance
Robust control
Optimal investment strategy
Utility function
Mispricing

ABSTRACT

This paper considers how to optimize reinsurance and investment decisions for an insurer who has aversion to model ambiguity, who wants to take into consideration time-varying investment conditions via mean reverting models, and who wants to take advantage of statistical arbitrage opportunities afforded by mispricing of stocks. We work under a complex realistic environment: The surplus process is described by a jump–diffusion model and the financial market contains a market index, a risk-free asset, and a pair of mispriced stocks, where the expected return rate of the stocks and the mispricing follow mean reverting stochastic processes which take into account liquidity constraints. The insurer is allowed to purchase reinsurance and to invest in the financial market. We formulate an optimal robust reinsurance-investment problem under the assumption that the insurer is ambiguity-averse to the uncertainty from the financial market and to the uncertainty of the insured's claims. Ambiguity aversion is an aversion to the uncertainty taken by making investment decisions based on a misspecified model. By employing the dynamic programming approach, we derive explicit formulae for the optimal robust reinsurance-investment strategy and the optimal value function. Numerical examples are presented to illustrate the impact of some parameters on the optimal strategy and on utility loss functions. Among our various practical findings and recommendations, we find that strengthened market liquidity significantly increases the demand for hedging from the mispriced market, to take advantage of the statistical arbitrage it affords.

1. Introduction

Investments in financial markets provide an important mean for any insurer to increase profits from the surplus process, while reinsurance is key to helping the insurer avoid or transfer excessive risk. Investments and reinsurance are therefore rightfully attracting increasing attention from a growing number of insurance mathematics scholars, and are becoming a popular topic in the actuarial literature. For example, Schmidli (2002) considered the optimal reinsurance-investment problem of minimizing ruin probability; Bäuerle (2005) studied the optimal reinsurance problem with a mean–variance criterion; Bai and Guo (2008) focused on

☆ This research was supported by grants from the National Natural Science Foundation of China (Nos. 71501050, 71471045, 71201173, 71601055), the National Natural Science Foundation of China: the funds for creative research group (71721001), the Natural Science Foundation of Guangdong Province of China (Nos. 2017A030313399, 2017A030313397), the Innovation Team Project of Guangdong Colleges and Universities (No. 2016WCXTD012), the Innovative School Project in Higher Education of Guangdong Province of China (No. GWTP-GC-2017-03), the Humanities and Social Science Research Foundation of the National Ministry of Education of China (No. 15YJAZH051), and the U.S. National Science Foundation (Nos. DMS 1407762 and DMS 1734183).

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https://doi.org/10.1016/j.insmatheco.2018.03.004
0167-6687/

the optimal reinsurance-investment problem by maximizing exponential utility with a no-shorting constraint; Zeng and Li (2011) considered the optimal time-consistent reinsurance-investment problem with mean variance criterion; Gu et al. (2012) investigated the optimal excess-of-loss reinsurance and investment problem with a Constant-Elasticity-of-Variance model.

However, most papers listed above assume the risky asset's appreciation rate is a constant or a deterministic function. This goes against empirical stock market data as widely understood and noted by many authors (see for instance Merton (1980) and Rapach and Zhou (2013)): in reality, appreciation rates are not constant, and more specifically, mean rates of return are more appropriately modeled as being stochastic processes; they are thus often referred to as *stochastic risk premia*. Typically, stochastic risk premium processes are described by mean-reverting (MR) models, which are given in the empirical literature on real market, see Chapter 20 in Cochrane (2001) and Rapach and Zhou (2013); they play an important role in the portfolio by providing for the possibility of time-varying investment opportunities. Thus, a MR risk premium is seen as a valuable feature of the risky asset's price. Baev and Bondarev (2007), who discussed the ruin probability of an insurer, studied the optimal investment problem for a risky model in which the stock price has a stochastic risk premium given by an Ornstein–Uhlenbeck (OU) process, which is a special type of MR process. The OU process has the advantage of being amenable to explicit analyses in many cases because it is Gaussian. Liang et al. (2011) aimed to maximize the expected exponential utility of the terminal wealth where again the instantaneous rate of the stock's return follows an OU process; they obtained the optimal reinsurance and investment strategies. Gu et al. (2013) studied the optimal DC pension plan under an OU model. Recently, Yi et al. (2015a) studied dynamic portfolio selection with mean reversion describing the stochastic risk premium, under additional current specificities of Chinese and Hong Kong stock markets.

In this paper, we incorporate ambiguity aversion to study the optimal robust reinsurance-investment problem, where the stock's stochastic risk premium follows a MR process. At the moment, the use of MR risk premia is understudied in the implementation of robust optimization. Going back to the original work of Anderson et al. (2000) discussed below, and the seminar work of Maenhout (2004) on homothetic robustness, we know that robust optimization is helpful to account for the fact that drift coefficients are very difficult to pin down. Also, we describe several other papers above and below which use robustness this way, to give some flexibility on the inability to estimate a constant drift coefficient reliably. However, as stated above, if the real problem is that mean rates of return are difficult to estimate because they are actually stochastic, as in the case of stochastic risk premia, allowing for an uncertain constant drift parameter in an optimal investment problem does not capture the time-varying aspect of investment conditions in the models' specifications. In this sense, using a MR process for the risk premium is crucial for many stock markets where such a model feature is widely accepted (see Poterba and Summers (1988), Fouque et al. (2000), and Mukherji (2011) for empirical evidence). The paper by Yi et al. (2015a) about dynamic portfolio selection with MR stochastic risk premium takes this approach and combines it with an acceptance of modeling uncertainty, including for the parameters of the MR stochastic risk premium; they address this uncertainty by fully adopting robust optimization for all model drift parameters. They only consider applications to uncertainty in financial market, however. Our work picks up where they left off, by looking at the implications for insurance-reinsurance-investment problems. Moreover, we use jump-based and diffusive models, as is appropriate for distinguishing between event frequencies in the claims process and the market processes. Thus, based on Yi et al. (2015a), we incorporate market and claim stochasticity as well as modeling uncertainty for both the insurance model and the financial market model, and investigate an optimal robust reinsurance-investment strategy for the insurer, where the appreciation rate of the stocks and any possible arbitrage-inducing mispricing between two stock prices are described by MR processes. We analyze the impact of ambiguity aversion and mispricing on the optimal strategy.

We say a few more words in this introduction about ambiguity and about mispricing, and how our work incorporates these features in the reinsurance-investment problem under time-varying market conditions.

Ambiguity was developed as a way of addressing modeling uncertainty on the mean rates of return and other drift parameters in stochastic models for risky assets. Therefore, ambiguity has an important impact on investment decisions. This idea has been developed systematically as a method in quantitative investment finance for portfolio selection and asset pricing with model uncertainty or model misspecification. For example, Anderson et al. (2000) introduced ambiguity aversion into the Lucas model, and formulated alternative models. Uppal and Wang (2003) extended Anderson et al. (2000), and developed a framework which allows investors to consider the level of ambiguity. Maenhout (2004, 2006) optimized an inter-temporal consumption problem with ambiguity, and derived closed-form expressions for the optimal strategies. The resulting optimal decision schemes are legitimately called *Robust optimization* because they are highly robust to drift misspecifications. However these ideas should not be limited to financial risk modeling, and the same ambiguity exists in the expected surplus of insurers: some scholars have developed decision optimization under ambiguity to discuss the optimal reinsurance and investment problem. For instance, Zhang and Siu (2009) and Korn et al. (2012) used stochastic differential games to study the optimal reinsurance-investment problem with ambiguity; Yi et al. (2013, 2015a, b) studied the optimal proportional reinsurance-investment strategy with ambiguity aversion under expected utility maximization and mean–variance criterion. Li et al. (2017) focused on another kind of reinsurance, the excess-of-loss reinsurance, and discussed the reinsurance-investment problem with ambiguity aversion. Pun and Wong (2015) studied robust investment-reinsurance optimization with multiscale stochastic volatility. Zeng et al. (2016) studied the equilibrium strategy of a robust optimal reinsurance-investment problem under the mean–variance criterion. In all papers mentioned above, the (excess) mean rates of return of the risky assets are assumed to be constant. However, in real-world market as we mentioned, the appreciation rate of the risky asset changes with time and with market conditions.

In order to derive a more realistically implementable strategy, our work incorporates these time-varying conditions into our stochastic models for financial risk by using stochastic risk premia, and allows for mispricing between stocks prices, all the while studying the optimal reinsurance-investment question in the framework of model drift ambiguity and its robust optimal solutions.

Mispricing is a difference (discrepancy) between a pair of asset prices, where these prices describe assets or contingent claims which are identical or nearly identical. The asset/contingent-claim values ought to have the same or close to the same price during a same trading period but in reality they do not have the same price in different financial markets, because of the existence of frictions in markets which are not entirely mature. For a simple example, the stock of Agriculture Bank of China is traded on Chinese stock exchanges (Shanghai, Shenzhen) as shares A, and on the Hong Kong stock exchanges as shares H. Usually, the price of share A is different from that of share H. As explained in Gu et al. (2017), before 2015, China's trading policy did not permit individual traders to purchase share H. In 2015, the Chinese government opened up simultaneous investment in the mainland China and Hong Kong financial market, which means that a mainland China investor is allowed to invest in designated Hong-Kong stocks and vice-versa. Because the class of investors able to take advantage of this new situation is not universal, frictions remain, and price discrepancies persists to this day. This creates arbitrage opportunities for some Chinese investors. Under this condition, an insurer in China wishing to invest some of her surplus process into risky assets can typically make full use the price difference between share A and share H in her investment strategy. A common strategy to take advantage of this mispricing arbitrage is a "long-short" (L–S) strategy, which takes positions of equal size but opposite signs either in portfolio weight or in number of shares, see Liu and Longstaff (2004) and Jurek and Yang (2006). This type of strategy is used as long-term arbitrage opportunities, since it does not need to be modified or otherwise rebalanced over time, but that typically ignores temporary diversification benefits. Liu and Timmermann (2013) established a portfolio maximization framework to take full advantage of mispricing, showing that L-S is not always an optimal strategy.

In our paper, in the framework of maximizing the insurer's exponential utility with mispricing, we establish that our optimal investment strategy is no longer a pure L-S strategy, but rather consists of two parts: one part is the L-S strategy, the other part is what we call a common strategy, where the L-S strategy is not optimal if and only if the liquidities of the two stock prices are different. Our formulas show that the difference between the positions on the two markets increases linearly with the difference in liquidity parameters.

We adopt a relatively classical jump–diffusion model for our insurance surplus process; our financial market is diffusive only, and consists of the market index, one risk-free asset and a pair of stocks with mispricing, where the appreciation rate of the stocks and the mispricing are given by MR processes. Our goal is to maximize the expected exponential utility of the terminal wealth to find an optimal strategy and corresponding optimal value function, in as explicit a closed form as possible. This enables us to give numerical examples which show the impact of ambiguity aversion, the appreciation rate, mispricing, and reinsurance, on the optimal value function.

To summarize, compared to the existing literature, the contributions of this paper are twofold. First, we consider stochastic risk premia in the reinsurance and investment problem for an ambiguity-averse insurer (AAI), whose aversion to model ambiguity extends to the financial market, to the insurance business, and to the phenomenon of mispricing. As such, our paper is the first to allow the more realistic assumption of modeling uncertainty at all these levels simultaneously for the insurer–investor. Second, our AAI can take advantage of time-varying opportunities when investing in the financial markets, in the context of her utility-based preference to achieve an optimal investment and reinsurance strategy. Third, we believe our paper is only the second paper to study the joint impact of ambiguity aversion, mispricing, and reinsurance (this was first done in (Gu et al., 2017), but is the first to allow three different levels of ambiguity for modeling the diffusion uncertainty in finance and in insurance, and for modeling insurance jump certainty. In fact, this paper goes beyond the results of Gu et al. (2017) by working with models with features closer to real markets, where the insurance model is not independent of the financial market, and where the excess return of the stocks is no longer a constant, but follows a mean-reversion process to account for time-varying investment opportunities. Within this more complete modeling framework, some interesting results are given which cannot be obtained in the previous study. For instance, the optimal investment strategy is separated into two parts: one is a hedging strategy, and the other is common strategy. These allow us to provide explanations on how mispricing, liquidity, and excess returns affect the optimal investment strategy. Furthermore, our studies are illustrated by evaluating various utility losses which are specific to our framework. We find that for the sake of profits, mispricing and reinsurance cannot be ignored, if the AAI wants to avoid suffering large utility losses. Most notably, since we find that the utility loss increases significantly with the investment horizon, the effect of ignoring either mispricing or reinsurance will become compounded for anything longer than the shortest investment and business horizons. On the side of safety and/or caution, we find that it is to the insurer's significant advantage to take a more conservative strategy to avoid the sudden risk of working with a misspecified model when business and market conditions change. All these conclusions are fully quantitative.

The remainder of this paper is organized as follows. The financial market and the insurance model are described in Section 2. In Section 3 the optimal robust reinsurance-investment problem with mispricing is established and solved. Section 4 provides the numerical examples to illustrate our results and analyze the utility losses which come from ignoring ambiguity aversion, mispricing, or reinsurance, when managing an insurance surplus. Section 5 concludes this paper.

2. Economy and assumptions

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t, 0 \le t \le T\}, P)$ be a complete filtered probability space, in which $T > 0$ is a fixed constant, representing time horizon, $\{\mathcal{F}_t, 0 \le t \le T\}$ is a filtration, which describes the flow of information over time and the $\sigma-$algebra $\mathcal{F}_t$ describes the information available up to time t. We denote P as a reference measure and suppose that all stochastic processes given in the following are assumed to be adapted on this space.

2.1. Surplus process

Our insurer's surplus process is given by the classical Cramér–Lundberg (C–L) model with diffusion. In this jump–diffusion model, without reinsurance and investment, the surplus is given by

$$dR(t) = pdt - d\sum_{i=1}^{N_t} Y_i + \sigma_1 dB(t),$$

(see for example Bäuerle (2005)). This model has the following well-known features, which we discuss here for completeness:

- p is a positive constant, representing the rate at which premia are paid;
- $\{N_t, 0 \le t \le T\}$ is a Poisson process with intensity $\lambda > 0$, which represents the number of claims from time 0 to time t;
- Y_i is the size (cost) of the ith claim; the sequence of claim sizes $Y_1, Y_2, \ldots$ is a set of independent and identically distributed (i.i.d.) positive random variables; their distribution F is assumed to have finite first and second moments μ_∞ and σ_∞^2;
- It follows from the above that $\sum_{i=1}^{N_t} Y_i$ is a compound Poisson process; this represents the total value of all claims against the insurer in time interval $[0, t]$;
- The stochastic process $\{B(t), 0 \le t \le T\}$ is a standard Brownian motion independent of N, representing the diffusion risk of the surplus process;
- We assume that the premium rate p is computed based on the expected value principle with loading, which means that $p = (1+\eta)\lambda\mu_\infty$, where $\eta > 0$ is the relative safety loading of the insurer.

In this paper, the insurer needs to decide if she wants to purchase proportional reinsurance to decrease business risk, or acquire new business to increase her profits. In the case in which she chooses to purchase reinsurance, for each $t \in [0, T]$, she determines the quantity of reinsurance via the retention level, which is a proportion $q(t) \in [0, +\infty)$. When $q(t) \in (0, 1]$, this means she purchases proportional reinsurance. Then for each claim, the insurer only pays 100q% while the rest 100(1-q)% is paid by the reinsurer. This reinsurance has a cost, and we assume that the insurer diverts part of each premium to the reinsurer. Since in this paper we assume that the insurer receives premia continuously over time, the same is assumed for the reinsurance premia paid to the reinsurer. When $q(t) \in (1, \infty)$, since the insurer is retaining more than 100q% of risk, we interpret this by saying that she acquires new business from the other insurers as a reinsurer herself. Overall, we assume that reinsurance is not inexpensive, i.e., the reinsurer's safety loading θ is greater than the insurer's safety loading η. We use the expected value principle with loading to compute the reinsurance premium. This implies that the reinsurance premium rate must be $(1 - q(t))(1 + \theta)\lambda\mu_\infty$. The reinsurance strategy can thus be assimilated to the stochastic process $\{q(t) : t \in [0, T]\}$ and one can easily show that the insurer's resulting surplus process satisfies the following stochastic dynamics:

$$\begin{aligned} dR(t) &= [(1+\theta)q(t)+\eta-\theta]\lambda\mu_\infty dt - q(t)d\sum_{i=1}^{N_t} Y_i + \sigma_1 dB(t) \\ &= [(1+\theta)q(t)+\eta-\theta]\lambda\mu_\infty dt + \sigma_1 dB(t) \\ &\quad - q(t)\int_{R^+} y(t)N(dt, dy), \end{aligned} \tag{1}$$

where $N(\cdot,\cdot)$ defined on $\Omega \times [0, T] \times R^+$ is the Poisson random measure corresponding to the Poisson process $\{N_t\}$. Denoting by $\nu(dt, dy) = \lambda dt dF(y)$, $E[\sum_{i=1}^{N_t} Y_i] = \int_0^t \int_{R^+} y\nu(dt, dy)$, this ν represents the compensator of the random measure $N(\cdot,\cdot)$. Therefore, the compensated measure $\widetilde{N}$ of the compound Poisson process $\sum_{i=1}^{N_t} Y_i$ is $\widetilde{N}(\cdot,\cdot) = N(\cdot,\cdot) - \nu(\cdot,\cdot)$. Finally, we can rewrite the insurer's surplus as:

$$\begin{aligned} dR(t) &= [(1+\theta)q(t)+\eta-\theta]\lambda\mu_\infty dt \\ &\quad - q(t)\int_{R^+} y(t)\widetilde{N}(dt, dy) - q(t)\int_{R^+} y(t)\lambda dt dF(y) \\ &\quad + \sigma_1 dB(t). \end{aligned}$$

2.2. Financial market

Our financial market consists of one risk-free asset, the market index, and a pair of stocks with mispricing. We will assume (see Eq. (2)) that the stocks' appreciation rate $a(t)$ is a stochastic variable following an MR process. The price process of the risk-free asset is expressed as the usual exponential function solving the elementary ordinary differential equation

$$\frac{dS_0(t)}{S_0(t)} = rdt,\ S_0(0) = s_0,$$

where $r > 0$ is the risk-free interest rate. The price process of the market index is expressed as

$$\frac{dP_m(t)}{P_m(t)} = (r + \mu_m)dt + \sigma_m dZ_m(t),$$

where the market risk premium μ_m and the market volatility σ_m are positive constants, and $\{Z_m(t)\}$ is a standard Brownian motion on $(\Omega, \mathcal{F}, P)$. The price processes of the pair of stocks satisfy the following the stochastic differential equations

$$\begin{aligned} \frac{dP_1(t)}{P_1(t)} &= (r + a(t))dt + \sigma dZ_t + bdZ_{1t} - l_1 X(t)dt,\ P_1(0) = P_{10}, \\ \frac{dP_2(t)}{P_2(t)} &= (r + a(t))dt + \sigma dZ_t + bdZ_{2t} + l_2 X(t)dt,\ P_2(0) = P_{20}, \end{aligned} \tag{2}$$

where l_1, l_2, σ, b are constant parameters. The term σdZ_t describes the common risk, the term bdZ_{it} describes the idiosyncratic risk of stock i, and $a(t)$ is the aforementioned premium (i.e., the excess return) for the common risk, whose MR dynamics are given by

$$da(t) = n(m - a(t))dt + \sigma_a dZ_a(t),\ a(0) = a_0, \tag{3}$$

where m is the long-run mean of the risk premium, and n is the degree of mean reversion (or mean-reversion rate). The term $l_i X(t)dt$ shows the effect of mispricing on the ith stock's price, where $X(t)$ is the pricing error or mispricing between two stocks, and is defined as

$$X(t) = \ln\frac{P_1(t)}{P_2(t)}.$$

Based on Eq. (2), using standard Itô's calculus, we find that the dynamics of the mispricing $X(t)$ satisfy the following equation

$$\begin{aligned} dX(t) &= -(l_1 + l_2)X(t)dt + bdZ_{1t} - bdZ_{2t} \\ &= (l_1 + l_2)(0 - X(t))dt + bdZ_{1t} - bdZ_{2t},\ X(0) = x_0. \end{aligned} \tag{4}$$

Thus Eq. (4) shows that X is also a MR process; it shows in particular that the long-run mean of the pricing error is 0, and its mean-reversion rate is $(l_1 + l_2)$. This also shows that investment opportunities are time-dependent. Note that l_1 and l_2 in Eq. (4) cannot equal zero at the same time. Otherwise, the pair of stocks would represent a pair with the same expected return rate but higher risk than the market index, which is not realistic. For the sake of convenience, we assume that $l_1 + l_2 > 0$, the same assumption is found in Liu and Timmermann (2013). Switching the direction of the inequality is equivalent to switching the roles of the two stocks. It is sometimes helpful to view l_1 and l_2 as liquidities. Indeed, mispricing typically occurs when markets contain frictions, which in turn can be due to inadequate liquidities, and in the case that the liquidities l_1 and l_2 are low, we will have a mispricing process $X(t)$ which takes longer to revert back to the zero mean, which is consistent with the idea that more friction accompanies increased illiquidity.

Note that ρ is the correlation coefficient between the Brownian motions $\{Z_t\}$ and $\{B(t)\}$, and ρ_0 is the correlation coefficient between $\{Z_m(t)\}$ and $\{Z_a(t)\}$. Moreover, there exist Brownian motions $\{\hat{B}(t)\}$ and $\{Z_0(t)\}$ satisfying the following equations:

$$dZ_t = \rho dB(t) + \hat{\rho} d\hat{B}(t),\ dZ_a(t) = \rho_0 dZ_m(t) + \hat{\rho}_0 dZ_0(t),$$

where $\rho, \rho_0 \in [-1, 1]$, $\hat{\rho} = \sqrt{1-\rho^2}$, and $\hat{\rho}_0 = \sqrt{1-\rho_0^2}$. Moreover, we assume all standard Brownian motions $\{Z_m(t)\}$, $\{B_t\}$, $\{\hat{B}(t)\}$, $\{Z_{1t}\}$, $\{Z_{2t}\}$ and $\{Z_0(t)\}$ are independent each other and all are independent of $N(dy, dt)$.

In addition to reinsurance, the insurer can invest in our financial market. Thus we denote by $u(t) = (\pi_m(t), \pi_1(t), \pi_2(t))$ the corresponding investment strategy, where $\pi_m(t)$ shows the amount of the wealth invested in the market index, and $\pi_1(t)$, and $\pi_2(t)$ denote the amounts invested in the two stocks, respectively, hence the remainder, $R(t) - \pi_m(t) - \pi_1(t) - \pi_2(t)$, is invested in the risk-free asset.

2.3. Wealth process

We denote the whole reinsurance-investment strategy by $\pi := \{\pi(t), 0 \le t \le T\} = \{(q(t), u(t)), t \in [0, T]\}$. As a result of adopting the strategy π, the insurer's corresponding reserve $\{W^\pi(t)\}_{0\le t\le T}$ satisfies the following stochastic dynamics:

$$\begin{aligned} dW^\pi(t) &= [W^\pi(t)r + \pi_m(t)\mu_m + a(t)(\pi_1(t) + \pi_2(t)) \\ &\quad + X(t)(\pi_2(t)l_2 - \pi_1(t)l_1)]dt \\ &\quad + \sigma(\pi_1(t) + \pi_2(t))(\rho dB(t) + \hat{\rho} d\hat{B}(t)) \\ &\quad + b(\pi_1(t)dZ_{1t} + \pi_2(t)dZ_{2t}) + \sigma_1 dB(t) \\ &\quad + \pi_m(t)\sigma_m dZ_m(t) + (\eta - \theta + (1+\theta)q(t))\lambda\mu_\infty dt \\ &\quad - q(t)\int_{R^+} y(t)N(dy, dt), \end{aligned} \tag{5}$$

where $W^\pi(0) = w_0$ and w_0 is the insurer's initial wealth.

3. Robust problem with mispricing

3.1. Utility function

In any classical reinsurance-investment quantitative setup, the insurer is assumed to be ambiguity-neutral in her investment decisions (i.e., is an ANI): this means that she is fully confident that the model determined by statistical estimation or calibration is the correct model; in this paper we refer to this model as the reference model. Under the so-called constant-absolute-risk-averse (CARA) utility assumption, the ANI maximizes her expected terminal wealth, which we thus express as the following optimization problem:

$$\max_{\pi\in\Pi_0} E_0^P[U(W^\pi(T))] = \max_{\pi\in\Pi_0} E_0^P\Big[-\frac{1}{\gamma}\exp\{-\gamma W^\pi(T)\}\Big], \tag{6}$$

where $\gamma > 0$ is the so-called absolute risk aversion coefficient, Π_0 is the set of admissible strategies which the ANI considers, and $E_t^P[\cdot] = E^P[\cdot|\mathcal{F}_t]$ is a short-hand notation for the conditional expectation at time t under the reference probability measure P, given all the available information up to that time.

Deviating from this traditional model, we discuss the optimization problem for the Ambiguity-Averse Insurer (AAI), who calls into question, or is skeptical about, the veracity of the reference model. In other words, the AAI does not have full confidence in the reference model, for instance because she is concerned about uncertainty on the parameters which may be due to a misspecification error. She recognizes that this model is only an approximation of reality and she wishes to take into consideration some alternative models, which preferably do not deviate very far from the reference model. We will have more to say below about what is meant by "too far" when we discuss the penalization term in the homothetic robustness framework. Every alternative model is characterized by a stochastic process ϵ (ϵ will be defined below) and the associated probability measure Q, which is equivalent to the reference measure P. This has the effect of allowing ambiguity on all drift parameters in the model. We denote this class of probability measures by $\mathcal{Q}$:

$$\mathcal{Q} = \{Q|Q \sim P\}.$$

According to the standard representation of the Radon–Nikodym derivative of any element Q of $\mathcal{Q}$ with respect to P, we know that to write down any such equivalent measure Q, there exists a progressively measurable process $\epsilon = (\varphi(t), \phi(t))$, such that

$$\frac{dQ}{dP} = \xi(T)$$

where $\xi(t)$ has the following form:

$$\begin{aligned}\xi(t) = \exp\Big\{&-\int_0^t \varphi(t)d\mathcal{Z}_t - \frac{1}{2}\int_0^t \|\varphi^2(t)\|dt\\ &-\int_0^t h(t)dB(t) - \frac{1}{2}\int_0^t h^2(t)dt\\ &+\int_0^t\int_0^\infty \ln\phi(t)N(dt,dy)\\ &+\int_0^t\int_0^\infty (1-\phi(t))\nu(dy,dt)\Big\}\end{aligned} \tag{7}$$

where $\varphi(t) = (h_m(t), \hat{h}(t), h_1(t), h_2(t), h_0(t))$, $\|\varphi(t)\|^2 = h_m^2(t) + \hat{h}^2(t) + h_1^2(t) + h_2^2(t) + h_0^2(t)$ and $d\mathcal{Z}_t = (dZ_m(t), d\hat{B}(t), dZ_{1t}, dZ_{2t}, dZ_0(t))$ where the various stochastic processes introduced here will have role revealed shortly below. As we mentioned, we are allowed to consider ambiguity in all drift parameters. In order to get results which are explicit and yield manageable expressions, we assume that, as far as the insurance model goes, the measure Q is different from the original measure P only in the intensity of the claim arrivals; we assume that the claim size distribution F is the same under both measures. Here it turns out that $\lambda\phi$ is the claim intensity of the Poisson process under the new measure Q. According to Girsanov's Theorem, the Brownian motions mentioned in our model can be defined under the equivalent measure Q as follows

$$\begin{aligned} &dZ_0^Q(t) = dZ_0(t) + h_0(t)dt, &&d\hat{B}_t^Q = d\hat{B}_t + \hat{h}(t)dt,\\ &dZ_{1t}^Q = dZ_{1t} + h_1(t)dt, &&dZ_{2t}^Q = dZ_{2t} + h_2(t)dt,\\ &dZ_m^Q(t) = dZ_m(t) + h_m(t)dt, &&dB^Q(t) = dB(t) + h(t)dt.\end{aligned} \tag{8}$$

Moreover, under the measure Q,

$$\widetilde{N}^Q(dt,dy) = N(dy,dt) - \lambda^Q dF(y)dt = N(dy,dt) - \lambda\phi(t)dF(y)dt$$

is a martingale. In the following definition for admissible strategies, reference is made to the worst-case measure Q^*. This measure will be defined as soon as we formulate the robust optimization problem, since it contains an infimum over all measures Q in $\mathcal{Q}$; we will see that this infimum is attained, denoting the argmin by Q^*. It is in this sense that the robust optimization problem can be solved using admissible strategies.

Definition 3.1. A strategy $\pi = \{q(t), \pi_m(t), \pi_1(t), \pi_2(t)\}_{t\in[0,T]}$ is said to be admissible, if

(1) $\forall t \in [0,T]$, $q(t) \in [0,\infty)$;

(2) π is predictable with respect to $\{\mathcal{F}_t\}$ and $E^{Q^*}[\int_0^T \|v(t)\|^2dt] < \infty$, where $\|v(t)\|^2 = q^2(t) + \pi_m^2(t) + \pi_1^2(t) + \pi_2^2(t)$;

(3) $\forall (t,w,x,a) \in [0,T]\times R\times R\times R$, Eq. (5) has a unique strong solution $\{W^\pi(t)\}_{t\in[0,T]}$ with $E_{t,w,x,a}^{Q^*}[U(W^\pi(t))] < \infty$, where Q^* is the probability measure in the worst case of our model.

Denote by Π the set of all admissible strategies. Similarly, we can obtain the set of admissible strategies Π_0, $\hat{\Pi}$ and $\tilde{\Pi}$ with no ambiguity aversion, no mispricing and no reinsurance, respectively.

Based on the admissible strategy $\pi\in\Pi$, the surplus process of the insurer under measure Q has the following stochastic dynamics:

$$\begin{aligned} dW^\pi(t) \;&= [W^\pi(t)r + a(t)(\pi_1(t)+\pi_2(t)) + X(t)(\pi_2(t)l_2\\ &\quad - \pi_1(t)l_1) + \pi_m(t)\mu_m\\ &\quad - \pi_m(t)\sigma_m h_m(t) - \sigma(\pi_1(t)+\pi_2(t))(\rho h(t)\\ &\quad + \hat{\rho}\hat{h}(t)) - b(\pi_1(t)h_1(t)\\ &\quad + \pi_2(t)h_2(t))]dt - \sigma_1 h(t)dt + \pi_m(t)\sigma_m dZ_m^Q(t)\\ &\quad + \pi_1(t)bdZ_{1t}^Q + \pi_2(t)bdZ_{2t}^Q\\ &\quad + \sigma_1 dB^Q(t) + (\pi_1(t)+\pi_2(t))\sigma(\rho dB^Q(t) + \hat{\rho}d\hat{B}^Q(t))\\ &\quad + [\eta - \theta + (1+\theta)q(t)]\lambda\mu_\infty dt\\ &\quad - q(t)\int_{R^+} y\big(\widetilde{N}^Q(dt,dy) + \lambda\phi(t)dF(y)dt\big).\end{aligned} \tag{9}$$

The idea of robust optimization starts with the realization that the reference model is only an approximation, determined by the ANI, of the true model. Therefore, the AAI wants to follow a strategy which might be robust to how the misspecified reference model might be compared with the true model, but even though she is suspicious of the reference model, she wants to avoid giving inordinate weight to alternatives which are far from her reference model, being reluctant to deviate too much from that model. Thus, in the process of setting the model, we use a measure of the distance between the reference model and the alternative models to penalize any deviations. Therefore, the alternative measures Q and their penalizations represent a trade-off between not being completely dependent on the reference model and, at the same time, not deviating from it too much. Using the surplus process (9) as the basis for a quantity to optimize, we formulate a robust control problem, inspired by Maenhout (2004), as follows:

$$V(t,w,x,a)=\sup_{\pi\in\Pi}\inf_{Q\in\mathcal{Q}}E^{Q}_{t,w,x,a}\left\{-\frac{\delta}{\gamma}e^{-\gamma W^{\pi}(T)}+\int_t^T\left(\frac{\frac{1}{2}\|\varphi(s)\|^2}{\psi_1(s)}+\frac{\frac{1}{2}h^2(s)}{\psi_2(s)}+\frac{(\phi(s)\ln\phi(s)-\phi(s)+1)\lambda}{\psi_3(s)}\right)ds\right\}, \tag{10}$$

where $E^{Q}_{t,w,x,a}=E^{Q}[\cdot|W(t)=w,X(t)=x,a(t)=a]$ and the second term in (10) represents the deviation from the reference measure, where $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ stand for preference parameters for the ambiguity aversion in modeling the financial market, the uncertainty of the insurance model, and the negative claim of the insurer. Typically, $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ are to be taken as measures of the information the insurer has about the true model; the greater $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ are, the less insurer knows about this model, the less faith she has in the reference model, and the more her worst case model will be allowed to deviate from the reference model. Note that $V(t,w,x,a)<0$ for the definition 3.1, which can be derived easily from Theorem 3 in Honda and Kamimura (2011). For convenience, similarly to Maenhout (2004), we assume that $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ are non-negative state-dependent functions having the following form

$$\psi_i(t)=-\frac{\alpha_i}{\gamma V(t,w,x,a)},\ i=1,2,3, \tag{11}$$

where $\alpha_1\geq 0,\alpha_2\geq 0$ and $\alpha_3\geq 0$ represent the insurer's ambiguity-aversion levels to the financial diffusion uncertainty, the insurance diffusion uncertainty, and the insurance claim jump uncertainty, respectively. When $\alpha_1=\alpha_2=\alpha_3\equiv 0$, the model (10) degenerates to the ANI model. Proposition 3.2 shows the corresponding results about this model.

In what follows in this section, we solve the optimization problem (10) and derive the optimal value function and the optimal strategy.

As we said, the AAI aims to obtain robustness by seeking the optimal strategy under a worst-case scenario. To be precise, for any fixed admissible strategy $\pi\in\Pi$, one seeks a worst case measure $Q^*(\pi)\in\mathcal{Q}$ which minimizes the utility for that strategy, and then an optimal robust strategy is attained by maximizing the resulting worst-case utility over all admissible strategies. This scheme translates mathematically into the following problem for exponential utility:

$$\sup_{\pi\in\Pi}E^{Q^*(\pi)}_{t,w,x,a}\left\{-\frac{1}{\gamma}e^{-\gamma W^{\pi}(T)}+\int_t^T\left(\frac{\frac{1}{2}\|\varphi(s)\|^2}{\psi_1(s)}+\frac{\frac{1}{2}h^2(s)}{\psi_2(s)}+\frac{(\phi(s)\ln\phi(s)-\phi(s)+1)\lambda}{\psi_3(s)}\right)ds\right\}.$$

Let $C^{1,2,2,2}([0,T]\times R\times R\times R)$ denote the class of functions which are continuously differentiable w.r.t. t on $[0,T]$, and twice continuously differentiable w.r.t. w,x,a on R, respectively. According to the robust Hamilton–Jacobi–Bellman equation (HJB for short, see Anderson et al. (2003), Maenhout (2006) and the properties of jump terms in such HJB (see, for example, Bäuerle (2005)), for any $J(t,w,x,a)\in C^{1,2,2,2}([0,T]\times R\times R\times R)$, we have

$$\sup_{\pi\in\Pi}\inf_{\varphi,\phi}\left\{\mathcal{A}^{\pi,\varphi,\phi}J(t,w,x,a)-\frac{\gamma J}{\alpha_1}\frac{1}{2}\|\varphi(t)\|^2-\frac{\gamma J}{\alpha_2}\frac{1}{2}h^2(t)-\frac{\gamma J\lambda}{\alpha_3}(\phi(t)\ln\phi(t)-\phi(t)+1)\right\}=0 \tag{12}$$

where

$$\begin{aligned}
\mathcal{A}^{\pi,\varphi,\phi}J(t,w,x,a)&=J_t+J_w(\eta-\theta+(1+\theta)q(t))\lambda\mu_\infty\\
&+J_w[wr+a(t)(\pi_1(t)+\pi_2(t))\\
&-\sigma(\rho h(t)+\hat{\rho}\hat{h}(t))(\pi_1(t)+\pi_2(t))\\
&-b(\pi_1(t)h_1(t)+\pi_2(t)h_2(t))\\
&+x(\pi_2(t)l_2-\pi_1(t)l_1)-\sigma_1h(t)+\pi_m(t)\mu_m-\pi_m(t)\sigma_mh_m(t)]\\
&+\frac{1}{2}J_{ww}\big[\sigma^2(\pi_1(t)+\pi_2(t))^2+b^2(\pi_1^2(t)\\
&+\pi_2^2(t))+\pi_m^2(t)\sigma_m^2+\sigma_1^2\\
&+2\sigma_1\sigma\rho(\pi_1(t)+\pi_2(t))\big]+J_{xx}b^2+\frac{1}{2}J_{aa}\sigma_a^2+J_{wa}\sigma_a\rho_0\pi_m(t)\sigma_m\\
&+J_{wx}b^2(\pi_1(t)-\pi_2(t))-J_x[(l_1+l_2)x+bh_1(t)-bh_2(t)]\\
&+J_a(n(m-a(t))-\sigma_a\rho_0h_m(t)-\sigma_a\hat{\rho}h_0(t))\\
&+\phi(t)\lambda E^{Q}[J(t,w-q(t)Y,x,a)-J(t,w,x,a)]
\end{aligned}$$

with the boundary condition $J(T,w,x,a)=-\frac{1}{\gamma}\exp\{-\gamma w\}$, where $J_t,J_w,J_{ww},J_x,J_{xx},J_{wx},J_a$ and J_{aa} represent the value function's partial derivative w.r.t. the corresponding variables.

In order to solve (12), we attempt an ansatz by assuming the solution $J(t,w,x,a)$ has the following form:

$$J(t,w,x,a)=-\frac{1}{\gamma}\exp\{-\gamma[P(t)w+\frac{1}{2}A_1(t)x^2+A_2(t)x+A_0(t)+\frac{1}{2}B_1(t)a^2+B_2(t)a+B_3(t)ax]\}. \tag{13}$$

From the boundary condition $J(T,w,x,a)=-\frac{1}{\gamma}\exp\{-\gamma w\}$, we have $P(T)=1$, $A_i(T)=0,B_j(T)=0,i=1,2,0,j=1,2,3$. A direct calculation yields the partial derivatives

$$\begin{aligned}
&J_t=-\gamma(P'(t)w+\frac{1}{2}A_1'(t)x^2+A_2'(t)x+A_0'(t)+\frac{1}{2}B_1'(t)a^2+B_2'(t)a+B_3'(t)ax)J,\\
&J_w=-\gamma P(t)J,\ J_{ww}=\gamma^2P(t)^2J,\\
&J_x=-\gamma J(A_1(t)x+A_2(t)+B_3(t)a),\\
&J_{xx}=(\gamma^2(A_1(t)x+A_2(t)+B_3(t)a)^2-A_1(t)\gamma)J,\\
&J_{wx}=\gamma^2P(t)(A_1(t)x+A_2(t)+B_3(t)a)J,\\
&J_{wa}=\gamma^2P(t)(B_1(t)a+B_2(t)+B_3(t)x)J,\\
&J_a=-\gamma(B_1(t)a+B_2(t)+B_3(t)x)J,\\
&J_{aa}=(\gamma^2(B_1(t)a+B_2(t)+B_3(t)x)^2-\gamma B_1(t))J\\
&\frac{J_w}{J_{ww}}=-\frac{1}{\gamma P(t)},\ \frac{J_{wx}}{J_{ww}}=\frac{A_1(t)x+A_2(t)+B_3(t)a}{P(t)},\\
&J(t,w-qY,x,a)-J(t,w,x,a)\\
&\quad=J(t,w,x,a)(\exp\{\gamma P(t)qY\}-1),
\end{aligned} \tag{14}$$

where $J=J(t,w,x,a)$.

According to the first-order conditions for $h_m(t)$, $\hat{h}(t)$, $h_1(t)$, $h_2(t)$, $h_0(t)$ and $h(t)$, we have

$$\begin{aligned}
&-J_w\pi_m(t)\sigma_m-J_a\sigma_a\rho_0-\frac{\gamma J}{\alpha_1}h_m=0,\\
&-J_w(\pi_1(t)+\pi_2(t))\sigma\hat{\rho}-\frac{\gamma J}{\alpha_1}\hat{h}(t)=0,\\
&-J_wb\pi_1(t)-J_xb-\frac{\gamma J}{\alpha_1}h_1(t)=0,\\
&-J_wb\pi_2(t)+J_xb-\frac{\gamma J}{\alpha_1}h_2(t)=0,\\
&-J_a\sigma_a\hat{\rho}_0-\frac{\gamma J}{\alpha_1}h_0(t)=0,\\
&-J_w(\sigma_1+(\pi_1(t)+\pi_2(t))\sigma\rho)-\frac{\gamma J}{\alpha_2}h(t)=0.
\end{aligned}$$

By solving the equations above, it follows that

$$\begin{aligned}
h_m^*(t) &= -\frac{\alpha_1}{\gamma J}(J_w\pi_m(t)\sigma_m + J_a\sigma_a\rho_0) = \alpha_1(\pi_m(t)\sigma_m P(t) \\
&\quad + \sigma_a\rho_0(B_1(t)a + B_2(t) + B_3(t)x)); \\
\hat{h}^*(t) &= -\frac{\alpha_1}{\gamma J}(\pi_1(t)+\pi_2(t))\sigma\hat{\rho}J_w = \alpha_1\sigma\hat{\rho}P(t)(\pi_1(t)+\pi_2(t)); \\
h_1^*(t) &= -\frac{\alpha_1}{\gamma J}b(\pi_1(t)J_w + J_x) = \alpha_1 b(\pi_1(t)P(t) \\
&\quad + A_1(t)x + A_2(t) + B_3(t)a); \\
h_2^*(t) &= -\frac{\alpha_1}{\gamma J}b(\pi_2(t)J_w - J_x) = \alpha_1 b(\pi_2(t)P(t) \\
&\quad - A_1(t)x - A_2(t) - B_3(t)a); \\
h_0^*(t) &= -\frac{\alpha_1}{\gamma J}J_a\sigma_a\hat{\rho}_0 = \alpha_1\sigma_a\hat{\rho}_0(B_1(t)a + B_2(t) + B_3(t)x); \\
h^*(t) &= -\frac{\alpha_2}{\gamma J}(\sigma_1 + (\pi_1(t)+\pi_2(t))\sigma\rho)J_w \\
&= \alpha_2 P(t)(\sigma_1 + (\pi_1(t)+\pi_2(t))\sigma\rho).
\end{aligned} \tag{15}$$

Similarly, by the first-order condition for $\phi(t)$, i.e.,

$$\lambda E^P[J(t, w - q(t)Y, x, a) - J(t, w, x, a)] - \frac{1}{\alpha_3}\gamma J(t, w, x, a)\lambda \ln\phi(t) = 0,$$

which gives

$$\begin{aligned}
\phi^*(t) &= \exp\{\frac{\alpha_3}{\gamma J(t, w, x, a)}E^P[J(t, w - q(t)Y, x, a) - J(t, w, x, a)]\} \\
&= \exp\{\frac{\alpha_3}{\gamma}E^P[e^{P(t)q(t)\gamma Y} - 1]\}.
\end{aligned} \tag{16}$$

Therefore, we obtain the worst case measure Q^* which is determined by $\epsilon^* = (\varphi^*, h^*, \phi^*)$ and $\varphi^* = (h_m^*, \hat{h}^*, h_1^*, h_2^*, h_0^*)$.

In the following, we seek the optimal robust reinsurance-investment strategy. Substituting (15) and (16) into the HJB equation (12), we have

$$\begin{aligned}
\sup_{\pi\in\Pi}\Big\{ & J_t + J_w\big[(\eta - \theta + (1+\theta)q(t))\lambda\mu_\infty \\
& + wr + x(\pi_2(t)l_2 - \pi_1(t)l_1) \\
& + a(\pi_1(t)+\pi_2(t)) + \pi_m(t)\mu_m\big] \\
& + \frac{1}{2}J_{ww}\big[\pi_m^2(t)\sigma_m^2 + (\pi_1^2(t)+\pi_2^2(t))b^2 + \sigma_1^2 \\
& + (\pi_1(t)+\pi_2(t))^2\sigma^2 + 2\sigma_1\sigma\rho(\pi_1(t)+\pi_2(t))\big] \\
& + J_{xx}b^2 - J_x(l_1+l_2)x \\
& + J_{wx}b^2(\pi_1(t)-\pi_2(t)) + J_a n(m-a) + \frac{1}{2}J_{aa}\sigma_a^2 \\
& + J_{wa}\sigma_a\rho_0\pi_m(t)\sigma_m \\
& + \frac{\alpha_1}{2\gamma J}(J_w\pi_m(t)\sigma_m + J_a\sigma_a\rho_0)^2 \\
& + \frac{\alpha_1}{2\gamma J}J_a^2\sigma_a^2\hat{\rho}_0^2\frac{\alpha_1\sigma^2\hat{\rho}^2J_w^2(\pi_1(t)+\pi_2(t))^2}{2\gamma J} \\
& + \frac{\alpha_1 b^2(J_x + J_w\pi_1(t))^2}{2\gamma J} + \frac{\alpha_1 b^2(J_x - J_w\pi_2(t))^2}{2\gamma J} \\
& + \frac{\alpha_2 J_w^2(\sigma_1 + (\pi_1(t)+\pi_2(t))\sigma\rho)^2}{2\gamma J} \\
& + \frac{\gamma J\lambda}{\alpha_3}(\phi^*(t) - 1)\Big\} = 0.
\end{aligned} \tag{17}$$

Differentiating Eq. (17) w.r.t. $q(t)$ implies

$$J_w(1+\theta)\lambda\mu_\infty + \frac{1}{\alpha_3}\gamma J\lambda\frac{\partial\phi^*(t)}{\partial q(t)} = 0.$$

According to (14) and (16), we know that the optimal reinsurance strategy $q^*(t)$ satisfies the following equation

$$\begin{aligned}
&(1+\theta)\mu_\infty \\
&= \exp\{\frac{\alpha_3}{\gamma}E^P[e^{P(t)q^*(t)\gamma Y(t)} - 1]\}E^P[Y(t)e^{P(t)q^*(t)\gamma Y(t)}].
\end{aligned} \tag{18}$$

We assert that $q^*(t) > 0$. If this were not true, then $e^{P(t)q^*(t)\gamma Y(t)} < 1$ and

$$\begin{aligned}
&\exp\{\frac{\alpha_3}{\gamma}E^P[e^{P(t)q^*(t)\gamma Y(t)} - 1]\}E^P[Y(t)e^{P(t)q^*(t)\gamma Y(t)}] \\
&< E^P[Y(t)] = \mu_\infty < (1+\theta)\mu_\infty.
\end{aligned}$$

This would cause a contradiction with (18).

According to the first-order conditions for $\pi_m(t)$, $\pi_1(t)$ and $\pi_2(t)$, we have

$$\begin{aligned}
\pi_m^*(t) &= e^{-r(T-t)}[\frac{\mu_m}{\sigma_m^2(\alpha_1+\gamma)} - \frac{\sigma_a\rho_0}{\sigma_m}(B_1(t)a + B_2(t) + B_3(t)x)], \\
\pi_1^*(t) &= \frac{1}{P(t)}[(\frac{1}{K}a) + (\frac{l_2-l_1}{2K} - \frac{l_1+l_2}{2(\alpha_1+\gamma)b^2})x \\
&\quad - (A_1(t)x + A_2(t) + B_3(t)a)] - \frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K}, \\
\pi_2^*(t) &= \frac{1}{P(t)}[(\frac{1}{K}a) + (\frac{l_2-l_1}{2K} + \frac{l_1+l_2}{2(\alpha_1+\gamma)b^2})x \\
&\quad + (A_1(t)x + A_2(t) + B_3(t)a)] - \frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K}
\end{aligned}$$

where $K = 2\sigma^2(\gamma + \alpha_1\hat{\rho}^2 + \alpha_2\rho^2) + b^2(\alpha_1+\gamma)$.

Inserting $q^*(t)$ and $\pi_m^*(t)$, $\pi_1^*(t)$, $\pi_2^*(t)$ into (17) and identifying the coefficient of the monomials w and x^2, x, a^2, a, ax as zero, we have

$$\gamma P'(t) + \gamma P(t)r = 0; \tag{19}$$

$$\begin{aligned}
&-\frac{1}{2}A_1'(t) + \frac{1}{2}\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2B_3^2(t) \\
&- \frac{1}{4}(\frac{(l_2-l_1)^2}{K} + \frac{(l_1+l_2)^2}{(\alpha_1+\gamma)b^2}) = 0,
\end{aligned} \tag{20}$$

$$\begin{aligned}
&-A_2'(t) + (\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_2(t)B_3(t) \\
&+ (\frac{\mu_m\sigma_a\rho_0}{\sigma_m} - nm)B_3(t) + \frac{\alpha_2\sigma_1\sigma\rho P(t)(l_2-l_1)}{K} = 0,
\end{aligned} \tag{21}$$

$$-\frac{1}{2}B_1'(t) + \frac{1}{2}\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2B_1^2(t) + nB_1(t) - \frac{1}{K} = 0, \tag{22}$$

$$\begin{aligned}
&-B_2'(t) + \sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2B_1(t)B_2(t) + nB_2(t) \\
&+ (\frac{\mu_m\sigma_a\rho_0}{\sigma_m} - nm)B_1(t) + \frac{2(\alpha_2+\gamma)\sigma_1\sigma\rho P(t)}{K} \\
&- \frac{2b^2(\alpha_1+\gamma)\gamma\sigma_1\sigma\rho P(t)}{K^2} = 0,
\end{aligned} \tag{23}$$

$$-B_3'(t) + nB_3(t) + (\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(t)B_3(t) - \frac{l_2-l_1}{K} = 0, \tag{24}$$

$$\begin{aligned}
&-A_0'(t) + \frac{1}{2}(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_2^2(t) - b^2A_1(t) \\
&+ (\frac{\mu_m\sigma_a\rho_0}{\sigma_m} - nm)B_2(t) - \frac{1}{2}\sigma_a^2B_1(t) \\
&- \frac{\mu_m^2}{2\sigma_m^2(\alpha_1+\gamma)} \\
&+ \frac{(\alpha_2+\gamma)(\gamma-\alpha_2)\sigma_1^2\sigma^2\rho^2P^2(t)}{K} + \frac{1}{2}\sigma_1^2(\alpha_2+\gamma)P^2(t) \\
&- (\eta - \theta + (1+\theta)q^*(t))\lambda\mu_\infty P(t) \\
&+ \frac{\lambda}{\alpha_3}(\phi^*(t) - 1) = 0.
\end{aligned} \tag{25}$$

Taking into account the boundary conditions $P(T)=1$ and $A_i(T)=0, i=0,1,2, B_j(T)=0, j=1,2,3$, and solving the differential equations (19)–(25) we obtain the explicit expressions of $P(t)$, $A_1(t)$, $A_2(t)$, $A_0(t)$, $B_1(t)$, $B_2(t)$ and $B_3(t)$ as follows

$$\begin{aligned}
P(t) &= e^{r(T-t)},\\
A_1(t) &= -\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2\int_t^T B_3^2(s)ds\\
&\quad+\frac{T-t}{2}\Big(\frac{(l_2-l_1)^2}{K}+\frac{(l_1+l_2)^2}{(\alpha_1+\gamma)b^2}\Big),\\
A_2(t) &= -\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2\int_t^T B_2(s)B_3(s)ds\\
&\quad-\Big(\frac{\mu_m\sigma_a\rho_0}{\sigma_m}-nm\Big)\int_t^T B_3(s)ds\\
&\quad-\frac{\alpha_2\sigma_1\sigma\rho P(t)(l_2-l_1)}{K}(T-t),\\
A_0(t) &= -\frac{(\alpha_1+\gamma)\sigma_a^2}{2}\hat{\rho}_0^2\int_t^T B_2^2(s)ds+b^2\int_t^T A_1(s)ds\\
&\quad-\Big(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-nm\Big)\int_t^T B_2(s)ds\\
&\quad+\frac{\sigma_a^2}{2}\int_t^T B_1(s)ds+\int_t^T\Big(\frac{\mu_m^2}{2\sigma_m^2(\alpha_1+\gamma)}\\
&\quad-\frac{(\alpha_2+\gamma)(\gamma-\alpha_2)\sigma_1^2\sigma^2\rho^2P^2(t)}{K}\\
&\quad-\frac{1}{2}\sigma_1^2(\alpha_2+\gamma)P^2(t)\Big)dt\\
&\quad+\lambda\mu_\infty\int_t^T(\eta-\theta+(1+\theta)q^*(s))e^{r(T-s)}ds\\
&\quad-\frac{\lambda}{\alpha_3}\int_t^T(\phi^*(s)-1)ds,
\end{aligned}\tag{26}$$

$$\begin{aligned}
B_1(t) &= \frac{e^{-\epsilon_1(T-t)}-e^{-\epsilon_2(T-t)}}{\Big(\frac{e^{-\epsilon_2(T-t)}}{\epsilon_2}-\frac{e^{-\epsilon_1(T-t)}}{\epsilon_1}\Big)(\alpha_1+\gamma)\hat{\rho}_0^2\sigma_a^2},\\
B_2(t) &= -e^{-\int_t^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\int_t^T\Big(\frac{\mu_m\sigma_a\rho_0}{\sigma_m}-nm\Big)B_1(s)\\
&\quad\times e^{\int_s^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(u)du}ds,\\
&\quad-e^{-\int_t^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\int_t^T\frac{2\sigma_1\sigma\rho P(s)}{K}(\alpha_2+\gamma)\\
&\quad\times e^{\int_s^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(u)du}ds,\\
&\quad+e^{-\int_t^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\int_t^T\frac{2\sigma_1\sigma\rho P(s)b^2(\alpha_1+\gamma)\gamma}{K^2}\\
&\quad\times e^{\int_s^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(u)du}ds,\\
B_3(t) &= e^{-\int_t^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\\
&\quad\times\int_t^T\frac{l_2-l_1}{K}e^{\int_s^T(n+(\alpha_1+\gamma)\sigma_a^2\hat{\rho}_0^2B_1(u)du}ds,
\end{aligned}\tag{27}$$

where $\epsilon_1=n+\sqrt{(n^2+\frac{2\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2}{K})}$ and $\epsilon_2=n-\sqrt{(n^2+\frac{2\sigma_a^2(\alpha_1+\gamma)\hat{\rho}_0^2}{K})}$.

Therefore, the optimal investment strategy can be given by

$$\begin{aligned}
\pi_m^*(t) &= e^{-r(T-t)}\Big[\frac{\mu_m}{\sigma_m^2(\alpha_1+\gamma)}-\frac{\sigma_a\rho_0}{\sigma_m}(B_1(t)a+B_2(t)+B_3(t)x)\Big],\\
\pi_1^*(t) &= e^{-r(T-t)}\Big[\Big(\frac{1}{K}a\Big)+\Big(\frac{l_2-l_1}{2K}-\frac{l_1+l_2}{2(\alpha_1+\gamma)b^2}\Big)x\\
&\quad-(A_1(t)x+A_2(t)+B_3(t)a)\Big]-\frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K},\\
\pi_2^*(t) &= e^{-r(T-t)}\Big[\Big(\frac{1}{K}a\Big)+\Big(\frac{l_2-l_1}{2K}+\frac{l_1+l_2}{2(\alpha_1+\gamma)b^2}\Big)x\\
&\quad+(A_1(t)x+A_2(t)+B_3(t)a)\Big]-\frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K}.
\end{aligned}\tag{28}$$

Evidently, the amounts invested in the financial market are linear function of x and a. Moreover, the optimal strategy invested in the stocks can be divided into two parts for each stock. Let

$$\begin{aligned}
\pi_{L-S}^*(t) &= e^{-r(T-t)}\Big[\frac{l_1+l_2}{2(\alpha_1+\gamma)b^2}x\\
&\quad+(A_1(t)x+A_2(t)+B_3(t)a)\Big],
\end{aligned}\tag{29}$$

and

$$\pi_{CM}^*(t)=e^{-r(T-t)}\Big[\frac{1}{K}a+\frac{l_2-l_1}{2K}x\Big]-\frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K}.\tag{30}$$

Then, we have

$$\pi_1^*(t)=\pi_{CM}^*(t)-\pi_{L-S}^*(t),\ \pi_2^*(t)=\pi_{CM}^*(t)+\pi_{L-S}^*(t).\tag{31}$$

This expression (31) means that the optimal investment strategy invested in the two stocks is no longer a pure L-S strategy. Instead, it comprises two parts, one part (i.e., $-\pi_{L-S}^*(t)$ for stock 1 and $(\pi_{L-S}^*(t))$ for stock 2) is the L-S strategy, the other part (i.e., $\pi_{CM}^*(t)$ for both stocks 1 and 2) is the common strategy. In particular, when $l_1=l_2$, i.e., when stock 1 and stock 2 have the same liquidity, we still have non-zero common strategy, but the long–short strategy π_{L-S}^* only depend on the mispricing x and is independent of the appreciation rate a; and the common strategy π_{CM}^* depends only on the appreciation rate and is independent of the mispricing. This can be interpreted as saying that, under a balanced market with friction, in order to take advantage of a mispricing arbitrage opportunity, the part of the strategy which confers that arbitrage advantage (L-S) need only concern itself with the mispricing level, not the non-stationarity of appreciation rates, while the part of the strategy which takes advantage of risk diversification (common), not arbitrage, only needs to worry about appreciation rates. This is a rather natural and intuitive situation, but it is interesting that this intuition breaks down when liquidities are unbalanced. More details and numerical analysis on π_{L-S}^* and π_{CM}^* will be given in Section 4.

Summarizing the discussion above and using a verification theorem whose proof is straightforward (see Yi et al. (2015a), Mataramvura and Øksendal (2008) and Honda and Kamimura (2011), we have the following Theorem.

Theorem 3.1. *For the optimization problem (10), the value function $V(t,w,x,a)=J(t,w,x,a)$ if $J(t,w,x,a)$ is the solution of (12), i.e.,*

$$\begin{aligned}
V(t,w,x,a) &= -\frac{1}{\gamma}\exp\Big\{-\gamma\Big[e^{r(T-t)}w+\frac{1}{2}A_1(t)x^2+A_2(t)x\\
&\quad+A_0(t)+\frac{1}{2}B_1(t)a^2+B_2(t)a+B_3(t)ax\Big]\Big\},
\end{aligned}$$

where $A_1(t)$, $A_2(t)$, $A_0(t)$, $B_1(t)$, $B_2(t)$ and $B_3(t)$ are given by (26) and (27); the corresponding optimal reinsurance-investment strategy is given by

$$\pi^*=(q^*(t),\pi_m^*(t),\pi_1^*(t),\pi_2^*(t)),$$

where $q^*(t)$ is determined by (18), $\pi_m^*(t)$, $\pi_1^*(t)$ and $\pi_2^*(t)$ are given by (28).

The optimal reinsurance strategy is dependent on both the ambiguity aversion α_3 and the absolute risk aversion coefficient γ.

The optimal investment strategy is driven by the sum $\alpha_1+\gamma$, the ambiguity-aversion level α_2, and depends on the mispricing x and the appreciation rate $a(t)$ of the risky asset. One might legitimately wonder why the investment finance strategy requires knowledge of the ambiguity level from the insurance model. This is due to the fact that the Brownian motions $\{B(t)\}$ and $\{Z(t)\}$ are correlated. The insurer has to consider the risk from the financial market and from the surplus process when she makes investment decisions. The amount invested in the two stocks will decrease with the increase of ambiguity aversion α_1 and α_2. Note that the optimal strategy invested in both stocks is not an L-S strategy even in the case $l_1=l_2$, because π_{CM} does not equal 0 as we mentioned: it changes with the parameters a and $\alpha_1,\ \alpha_2,\gamma$ and others. This persists even when liquidities are equal and correlations are zero, because of the presence of the term a/K.

If the insurer is an ANI, the optimization problem degenerates into optimization problem (6) without ambiguity aversion. Similarly to Theorem 3.1, we have the following Proposition.

Proposition 3.2. *For optimization problem (6), the value function is given by*

$$\begin{aligned}V^0(t,w,x,a)=&-\frac{1}{\gamma}\exp\{-\gamma[P(t)w+\frac{1}{2}A_{10}(t)x^2\\&+A_{20}(t)x+A_{00}(t)+\frac{1}{2}B_{10}(t)a^2\\&+B_{20}(t)a+B_{30}(t)ax]\},\end{aligned}$$

and the optimal strategy $\pi_0^*(t)=\pi^*(t)|_{\alpha_1=\alpha_2=0}$, where

$$\begin{aligned}A_{10}(t)=&-\sigma_a^2\gamma\hat{\rho}_0^2\int_t^T B_3^2(s)ds\\&+\frac{T-t}{2}(\frac{(l_2-l_1)^2}{(b^2+2\sigma^2)\gamma}+\frac{(l_1+l_2)^2}{\gamma b^2}),\\
A_{20}(t)=&-\sigma_a^2\gamma\hat{\rho}_0^2\int_t^T B_2(s)B_3(s)ds-(\frac{\mu_m\sigma_a\rho_0}{\sigma_m}-nm)\int_t^T B_3(s)ds,\\
B_{20}(t)=&-e^{-\int_t^T(n+\gamma\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\int_t^T[(\frac{\mu_m\sigma_a\rho_0}{\sigma_m}-nm)B_1(s)\\&\times e^{\int_s^T(n+\gamma\sigma_a^2\hat{\rho}_0^2B_1(u))du}\\&+\frac{4\sigma_1\sigma^3\rho e^{r(T-s)}}{(2\sigma^2+b^2)^2}]ds,\\
B_{30}(t)=&-e^{-\int_t^T(n+\gamma\sigma_a^2\hat{\rho}_0^2B_1(s))ds}\int_t^T\frac{l_2-l_1}{(b^2+2\sigma^2)\gamma}\\&\times e^{\int_s^T(n+\gamma\sigma_a^2\hat{\rho}_0^2B_1(u))du}ds,\\
A_{00}(t)=&-\frac{\gamma\sigma_a^2}{2}\hat{\rho}_0^2\int_t^T B_2^2(s)ds+b^2\int_t^T A_1(s)ds\\&-(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-nm)\int_t^T B_2(s)ds\\&+\frac{\sigma_a^2}{2}\int_t^T B_1(s)ds+\int_t^T(\frac{\mu_m^2}{2\sigma_m^2\gamma}-\frac{1}{2}\sigma_1^2\gamma P^2(t)\\&-\frac{\sigma_1^2\sigma^2\rho^2\gamma P^2(t)}{2\sigma^2+b^2})dt\\&+\lambda\mu_\infty\int_t^T(\eta-\theta+(1+\theta)q^*(s))e^{r(T-s)}ds\\&+\lambda E^Q[e^{P(t)q_0(t)\gamma Y}-1],\\
B_{10}(t)=&\frac{e^{-\epsilon_1(T-t)}-e^{-\epsilon_2(T-t)}}{(\frac{e^{-\epsilon_2(T-t)}}{\epsilon_2}-\frac{e^{-\epsilon_1(T-t)}}{\epsilon_1})\gamma\hat{\rho}_0^2\sigma_a^2},\end{aligned}\tag{32}$$

with $\epsilon_1=n+\sqrt{(n^2+\frac{2\sigma_a^2}{2\sigma^2+b^2})}$ and $\epsilon_2=n-\sqrt{(n^2+\frac{2\sigma_a^2}{2\sigma^2+b^2})}$.

We point out that in this case the value function can be obtained by inserting $\alpha_1=\alpha_2=0$ into the value function of Theorem 3.1. We find that the utility function $V^0(t,w,x,a)>V(t,w,x,a)$, which means that ambiguity aversion causes the insurer to lose some utility, as well it should. This is the trade-off for being skeptical about one's modeling abilities and worrying about the consequences of misspecifying one's model. We believe that such skepticism is a common and healthy component of an insurance modeler's characteristics: the will to consider some modeling uncertainty when making decisions. Another way to interpret an AAI's willingness to give up some utility for this modeling peace of mind comes from the financial paradigm of risk and reward: lower ambiguity levels mean higher risk and should imply higher rewards on average. Under this interpretation, the ambiguity-averse model helps the AAI have a systematic way of considering some conservative strategies, accepting some utility loss to attain a safer strategy. In the next section we will provide some quantitatively explicit explanations and evaluations of these utility losses.

Before doing so, we must prepare the terrain by expressing optimal value functions and optimal strategies for an AAI who ignores mispricing and reinsurance in her business model. The idea is that one ought to be able to show that making full use of mispricing and reinsurance will benefit the insurer by achieving higher profits/utility.

(i) Case 1: No mispricing case. In this case, we assume that the AAI ignores the mispricing in the market between stock 1 and stock 2 and mistakes $\pi^{IM}=\pi^*|_{x=0}$ as the optimal strategy, i.e., $\pi^{IM}(t)=(q^*(t),u^{IM}(t))$, where $u^{IM}(t)=(\pi_m^{IM}(t),\pi_1^{IM}(t),\pi_2^{IM}(t))=(\pi_m^*(t),\pi_1^*(t),\pi_2^*(t))|_{x=0}$ are given by

$$\begin{aligned}\pi_m^{IM}(t)&=e^{-r(T-t)}[\frac{\mu_m}{\sigma_m^2(\alpha_1+\gamma)}-\frac{\sigma_a\rho_0}{\sigma_m}(B_1(t)a(t)+B_2(t))],\\
\pi_1^{IM}(t)&=e^{-r(T-t)}[\frac{1}{K}a(t)-(A_2(t)+B_3(t)a(t))]-\frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K},\\
\pi_2^{IM}(t)&=e^{-r(T-t)}[\frac{1}{K}a(t)+(A_2(t)+B_3(t)a(t))]-\frac{(\alpha_2+\gamma)\sigma_1\sigma\rho}{K}.\end{aligned}$$

We can obtain the corresponding value function V^{IM} by solving the following optimization problem

$$\begin{aligned}\inf_{Q^{IM}\in\mathcal{Q}}\Bigg\{&E^Q_{t,w,x,a}\Bigg[-\frac{1}{\gamma}\exp\{-\gamma W^{\pi^{IM}}(T)\}\\&+\int_t^T(\frac{R_1^{IM}(s)}{\psi_1^{IM}(s)}+\frac{(h^{IM}(s))^2}{\psi_2^{IM}(s)}+\frac{R_3^{IM}(s)}{\psi_3^{IM}(s)})ds\Bigg]\Bigg\},\end{aligned}\tag{33}$$

where

$$\begin{aligned}R_1^{IM}(s)&=\frac{1}{2}\|\varphi^{IM}(s)\|^2,\ R_3^{IM}(s)=(\phi^{IM}(s)\ln\phi^{IM}(s)-\phi^{IM}(s)+1)\lambda,\\
\psi_i^{IM}(s)&=-\frac{\alpha_i}{\gamma V^{IM}(s,w,x,a)},\ i=1,2,3,\varphi^{IM}(s)\\&=(h_m^{IM}(s),\hat{h}^{IM}(s),h_1^{IM}(s),h_2^{IM}(s)).\end{aligned}$$

$\psi_1^{IM}(s)$ and $\psi_2^{IM}(s)$ stand for preference parameters for ambiguity aversion to the financial market and the negative claims of the insurer. Q^{IM} is determined by (φ^{IM},ϕ^{IM}). Thus we can derive the following Proposition.

Proposition 3.3. *In optimization problem (33), where the insurer does not take advantage of the mispricing which exists in the financial market, the value function is given by*

$$\begin{aligned}V^{IM}(t,w,x,a)=&-\frac{1}{\gamma}exp\{-\gamma(P(t)w+\frac{1}{2}A_{1IM}(t)x^2\\&+A_{2IM}(t)x+A_{0IM}(t)+\frac{1}{2}B_{1IM}(t)a^2\\&+B_{2IM}(t)a+B_{3IM}(t)ax)\},\end{aligned}$$

where $P(t)$ is given by Theorem 3.1 and $A_{1IM}(t)$, $A_{2IM}(t)$, $A_{0IM}(t)$, $B_{1IM}(t)$, $B_{2IM}(t)$, $B_{3IM}(t)$ are determined by the following differential equations

$$-\frac{1}{2}A'_{1IM}(t)+(l_1+l_2)A_{1IM}(t)+(\alpha_1+\gamma)b^2A^2_{1IM}(t)$$
$$+\frac{1}{2}(\alpha_1+\gamma)\sigma_a^2B^2_{3IM}(t)=0,$$
$$-A'_{2IM}(t)+(l_1+l_2)(A_{2IM}(t)-A_2(t))$$
$$+\sigma_a^2(\alpha_1+\gamma)B_{3IM}(t)(B_{2IM}(t)-\rho_0^2B_2(t))$$
$$+2b^2(\alpha_1+\gamma)A_{1IM}(t)A_{2IM}(t)+(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-nm)B_{3IM}$$
$$-2b^2(\alpha_1+\gamma)A_2(t)A_{1IM}(t)$$
$$\frac{(\alpha_2+\gamma)(l_2-l_1)\sigma_1\sigma\rho P(t)}{K}=0,$$
$$-\frac{1}{2}B'_{1IM}(t)+nB_{1IM}(t)+\frac{1}{2}(\alpha_1+\gamma)\sigma_a^2(B^2_{1IM}(t)$$
$$+\rho_0^2B_1^2(t))+b^2(\alpha_1+\gamma)(B^2_{3IM}(t)+B_3^2(t))$$
$$-2b^2(\alpha_1+\gamma)B_{3IM}(t)B_3(t)-\sigma_a^2\rho_0^2(\alpha_1+\gamma)B_1(t)B_{1IM}(t)-\frac{1}{K}=0,$$

$$-B'_{2IM}(t)+nB_{2IM}(t)+(\alpha_1+\gamma)\sigma_a^2B_{1IM}(t)B_{2IM}(t)$$
$$+2b^2(\alpha_1+\gamma)A_{2IM}(t)B_{3IM}(t)$$
$$+(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-\sigma_a^2\rho_0^2(\alpha_1+\gamma)B_2(t))B_{1IM}(t)$$
$$-2b^2(\alpha_1+\gamma)(B_3(t)A_{2IM}(t)-A_2(t)B_{3IM}(t)$$
$$+A_2(t)B_3(t))+(\alpha_1+\gamma)\sigma_a^2\rho_0^2(B_1(t)B_2(t)$$
$$-B_1(t)B_{2IM}(t))+2b^2(\alpha_1+\gamma)A_2(t)B_3(t)$$
$$-nmB_{1IM}(t)+\frac{4\sigma_1\sigma\rho P(t)}{K}(\alpha_2+\gamma-\frac{\alpha_2\sigma^2(\gamma+\alpha_1\hat{\rho}^2+\alpha_2\rho^2)}{K})$$
$$-\frac{2(\alpha_2+\gamma)(\alpha_1+\gamma)\sigma_1\sigma\rho(\alpha_1+\gamma)P(t)b^2}{K^2}=0,$$
$$-B'_{3IM}(t)+(n+l_1+l_2)B_{3IM}(t)+(\alpha_1+\gamma)\sigma_a^2B_{1IM}(t)B_{3IM}(t)$$
$$+2b^2(\alpha_1+\gamma)A_{1IM}(t)B_{3IM}(t)$$
$$-2b^2(\alpha_1+\gamma)A_{1IM}(t)B_3(t)-\sigma_a^2\rho_0^2(\alpha_1+\gamma)B_1(t)B_{3IM}$$
$$-(l_1+l_2)B_3(t)-\frac{l_2-l_1}{K}=0,$$
$$-A'_{0IM}(t)+\frac{1}{2}\sigma_a^2(\alpha_1+\gamma)(B^2_{2IM}(t)+\rho_0^2B_2^2(t))$$
$$+b^2(\alpha_1+\gamma)(A^2_{2IM}(t)+A_2^2(t))$$
$$-b^2A_{1IM}(t)-\frac{1}{2}\sigma_a^2B_{1IM}(t)+(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-mn)B_{2IM}$$
$$-\sigma_a^2\rho_0^2(\alpha_1+\gamma)B_2(t)B_{2IM}(t)$$
$$-2b^2(\alpha_1+\gamma)A_2(t)A_{2IM}(t)-\frac{\mu_m^2}{\sigma_m^2(\alpha_1+\gamma)}$$
$$-\frac{(\alpha_2+\gamma)^2\sigma_1^2\sigma^2\rho^2P^2(t)}{K}$$
$$+\frac{\sigma_1^2P^2(t)}{2}(\alpha_1+\gamma)-P(t)(\eta-\theta+(1+\theta)q(t))\lambda\mu_\infty$$
$$+\frac{\lambda}{\alpha_3}(\phi^*(t)-1)=0.$$

In this case, there does exist mispricing in the financial market, and the AAI continues to invest in the two stocks even though the investment strategy ignores mispricing. So this case can be used to illustrate the importance of taking advantage of mispricing by defining the loss utility between the optimal strategy and the strategy where the AAI mistakenly believes $x=0$. We will investigate this further in Section 4.

(ii)Case 2: No reinsurance case. The insurer only invests in the financial market and does not purchase reinsurance, i.e., $\pi^{IR}=\pi^*|_{q^*(t)\equiv 1}$. The optimization problem (10) degenerates to investment-only problem. Setting $q^*(t)\equiv 1$ in Theorem 3.1, we easily derive the following Proposition.

Proposition 3.4. *For the optimization problem (10), if the insurer ignores reinsurance, the value function is given by*

$$V^{NR}(t,w,x,a)=-\frac{1}{\gamma}\exp\{-\gamma[P(t)w+\frac{1}{2}A_1(t)x^2$$
$$+A_2(t)x+A_{0NR}(t)+\frac{1}{2}B_1(t)a^2+B_2(t)a+B_3(t)ax]\},$$

where $A_1(t)$, $A_2(t)$, $B_1(t)$, $B_2(t)$, $B_3(t)$ is given by Theorem 3.1 and

$$A_{0NR}(t)=-\frac{(\alpha_1+\gamma)\sigma_a^2}{2}\hat{\rho}_0^2\int_t^TB_2^2(s)ds+b^2\int_t^TA_1(s)ds$$
$$-(\frac{\mu_m}{\sigma_m}\sigma_a\rho_0-nm)\int_t^TB_2(s)ds$$
$$+\frac{\sigma_a^2}{2}\int_t^TB_1(s)ds+\int_t^T(\frac{\mu_m^2}{2\sigma_m^2(\alpha_1+\gamma)}$$
$$-\frac{(\alpha_2+\gamma)(\gamma-\alpha_2)\sigma_1^2\sigma^2\rho^2P^2(s)}{K}$$
$$-\frac{1}{2}\sigma_1^2(\alpha_2+\gamma)P^2(s))ds+\lambda\mu_\infty\int_t^T(\eta+1)e^{r(T-s)}ds$$
$$-\frac{\lambda}{\alpha_3}\int_t^T(\exp\{\frac{\alpha_3}{\gamma}E^Q[e^{P(s)\gamma Y}-1]\}-1)ds.$$

4. Analysis of the results and numerical illustration

This section is devoted to providing some numerical examples to show the impact of the ambiguity aversion preference parameters $(\alpha_1,\alpha_2,\ \alpha_3)$, the mispricing $X(t)$, the risk premium $a(t)$, and some other parameters, on the AAI's optimal strategy and the utility loss functions. Our model's fixed parameters must be calibrated to the financial and insurance markets. We choose calibration against data relative to four Chinese bank stocks traded in both China and Hong Kong, as used in Table 1 of Yi et al. (2015a). Specifically, throughout this section, unless otherwise stated, the basic parameters of the financial market are given by $r=0.03,\ \gamma=0.8,\ \sigma=0.3,\ \ b=0.3,\ \lambda=10,\ l_1=0.2,\ l_2=0.6, \sigma_a=0.5;\ n=0.7;\ m=0;\ a=0.0132;\ \rho=0.5;\ \rho_0=0$. The other parameters are taken as $\eta=0.1,\ \theta=0.2,\ \ T=4,\ \beta=1.1,\ w=2,\ \alpha_1=\alpha_2=\alpha_3=0.2$, which are realistic values for our purposes. Moreover, we assume that the claim size Y_i follows the exponential distribution with parameter $\lambda_Y=6$ (an average of one claim every two months, corresponding to the idea of an insurer with a small number of large clients), and therefore the claim size density function is $f(y)=6e^{-6y}$.

4.1. Effects of relevant parameters on the robust optimal strategy

The parameters we concentrate on our analysis include the ambiguity aversion levels $(\alpha_1,\ \alpha_2,\ \alpha_3)$, the mean-reversion speeds $(n,\ l_1+l_2)$, and the intensity of claims arrival (λ). According to Theorem 3.1, we see that the optimal reinsurance strategy does not depend on the parameters given in the financial model; but the diffusion risk of the insurance model affects the optimal investment strategy because of the correlation between $\{B(t)\}$ and $\{Z_t\}$. We begin our analysis with the optimal investment strategy.

To fix ideas, we assume that the mispricing $X(t)>0$ and that $l_2>l_1>0$. This means that stock 1 will be overpriced and stock 2 will be underpriced, and the rate of reversion of the underpriced stock back to the mean is faster than the rate of reversion of the overpriced stock back to the mean. Thus an advantage of having a large long position in the underpriced stock will have a tendency to disappear faster. Therefore, one would expect the AAI to want to take advantage of mispricing today, as opposed to hoping that the mispricing will persist into the future. Other combinations of the relative positions of l_1 and l_2, and of X relative to 0, are possible, with corresponding interpretations and analyses, but we will not detail these herein, leaving such exercises for the interested reader.

According to (31), we saw that the optimal amounts invested in the two stocks are decomposed into two parts, i.e.,

$$\pi_1^*(t) = \pi_{CM}^*(t) - \pi_{L-S}^*(t),\ \pi_2^*(t) = \pi_{CM}^*(t) + \pi_{L-S}^*(t).$$

Recall that $\pi_{CM}^*(t)$ is called common strategy, which depends on the mispricing x, the appreciation rate a, and the ambiguity aversion parameters α_1 and α_2. Further, $\pi_{CM}^*(t)$ increases with respect to the mispricing x and the appreciation of the stock a, but decreases with respect to the ambiguity aversions α_1 and α_2. We notice that not only α_2 but also the correlation coefficient ρ affect the common strategy $\pi_{CM}(t)$: the smaller ρ is, the less impact the insurance model will have on the optimal strategy. The other part $\pi_{L-S}^*(t)$ represents the demand for hedging, it is not difficult to find that there will be more hedging demand with the increase of mispricing x and the appreciation rate a, because $A_1(t)$ and $B_3(t)$ are greater than 0 in our case. Moreover, the reversion speeds $l_1 + l_2$ and n of $X(t)$ and $a(t)$ have positive effects on the strategy π_{L-S}^*, i.e., higher liquidity in the financial market encourages larger positions in the hedging strategy. As Fig. 1 (a-b) shows that, in our case $x > 0$, when $a > 0$, π_{L-S}^* will increase with the increase of n and $l_1 + l_2$; on the other hand, π_{L-S}^* is not sensitive to n, but is very sensitive to $l_1 + l_2$. When $l_1 = l_2$, i.e., the stocks have the same liquidity, the hedging strategy does not depend on n. An intuitive explanation is that the hedging strategy comes from the difference in liquidity between the two stocks and their mispricing. Moreover, the speed of mean-reversion will push up or slow down the liquidity of the stocks, and therefore will have an effect on the hedging strategy. However, this impact will disappear at any time when the difference in liquidity between the two stocks is zero. These are also indications that the AAI should pay close attention to changes in liquidity when pursuing a strategy to exploit mispricing. The combined risk aversion parameter $\alpha_1 + \gamma$ is consistent with our intuition: π_{L-S}^* will decline with the increase of risk aversion. Moreover, as seen in Fig. 1, the time horizon (time to maturity) can have a major impact on the investment strategy. Specifically, Fig. 1 shows that π_{L-S}^* decreases as time t gets larger (as maturity approaches).

Now consider the effects, on the optimal investment strategies π_1^* and π_2^*, of the appreciation rate a, the mispricing x, and the ambiguity aversion to diffusion risk. As shown in Fig. 2, a has a positive effect on π_1^* and π_2^*: with the increase of a, the investor will invest more into stock 2, and decrease the absolute value in stock 1, and so is the sum $\pi_1^* + \pi_2^*$, which shows that the insurer will increase her position in stocks with the increase of a, as per standard intuition. The mispricing x has a positive impact on π_2^* and has a negative impact on π_1^*, i.e., with an increase in mispricing, the insurer will invest more into the underpriced stock 2, and short more of stock 1. As a result, she increases her amount $\pi_1^* + \pi_2^*$ with mispricing. If we fix a and x, as Fig. 3 shows, the insurer will decrease the positions in her two stocks under higher ambiguity aversion α_1 and α_2, and as maturity approaches, which causes the sum $\pi_1^* + \pi_2^*$ also to decrease with α_1 and α_2. In particular, Fig. 3 tells us that α_1 has a more significant impact on the optimal investment strategy than α_2, which shows that the uncertainty from the financial market takes a more important role in the investment activities.

Next, we discuss the impact of some parameters on the optimal market investment strategy π_m^*. From the first line of Eq. (28), we know that the amount invested in the market index has a positive relation with the market risk premium μ_m and a negative relationship with the market volatility σ_m. When the correlation coefficient $\rho_0 \neq 0$, the insurer will decrease the amount invested in the market index with the increase of appreciation rate a and mispricing x; this indicates that she would like to transfer more of her risky funds to the mispriced asset, to take advantage of that arbitrage.

In the following, we discuss the impact of some parameters on the optimal reinsurance strategy. As we mentioned above, the reinsurance strategy does not depend on the parameters in the financial model. So we only need to discuss the impact of the ambiguity aversion parameters (α_2, α_3) and the safety loading factor θ on the optimal reinsurance strategy $q^*(t)$.

The insurer buys reinsurance to avoid some of the risk from the claims. Therefore, one expects that the ambiguity aversion to the diffusion term in the insurance model should not affect the reinsurance strategy. This is indeed the case, since we can see that Eq. (18) does not depend on α_2, which accounts for ambiguity aversion to the insurance diffusive risk modeling. However, α_3 which represents the ambiguity aversion to claim risk modeling, has an important effect on the q^*. Fig. 4(a) shows that the AAI will decrease her retention level with the increase of ambiguity aversion α_3, i.e., the AAI prefers to buy more reinsurance and transfer more risk to the reinsurer if she is more ambiguity-averse to claims modeling. On the other hand, a higher safety loading factor θ imposed by the reinsurer will persuade the insurer to buy less reinsurance and to increase her retention level; see Fig. 4(b) for more details. For example, when $\theta = 0.45$, which represents a rather high level, the retention level is greater than 90%, which means the insurer tends not to buy reinsurance; she deems it too expensive in comparison with her investment and insurance business opportunities. Moreover, Fig. 4 illustrates the phenomenon by which, for short time horizon, the AAI tends to purchase more reinsurance and transfer more risk to the reinsurer, but for long time horizon, the AAI may transfer little to the reinsurer. This is consistent with the idea of becoming more conservative as one's investment and business opportunities approach maturity.

4.2. Effects of model parameters on the loss utility

In this subsection, we analyze the effects $(\alpha_1,\ \alpha_2,\ \alpha_3)$, l_1, l_2 and λ, on the loss utility functions, which are generated from ignoring ambiguity aversion, mispricing, and reinsurance.

As we explained earlier in the paper, the AAI is suspicious about the validity of her reference model, is averse to that ambiguity, and therefore she wants to find a conservative strategy which decrease the risk associated with model misspecification. On the contrary, the ANI believes fully in the reference model, and therefore pays more attention to maximizing her terminal utility, so she takes on more radical investment strategies. Therefore, as we mentioned, the ANI will always gain higher utility than the AAI, within the same given financial market and insurance models. We define the loss utility function

$$L^0 = 1 - \frac{V^0}{V}$$

where V and V^0 are given by Theorem 3.1 and Proposition 3.2, representing the value functions with and without ambiguity aversion, namely, representing the value functions for optimization problem (10) and optimization problem (6), respectively. As shown in Fig. 5, the utility loss from ignoring the ambiguity will increase with the increase of ambiguity aversion $\alpha_1,\ \alpha_2$ and α_3, and is more sensitive to them with the increase of liquidity, see Fig. 5(a) with $l_1 = 0.2,\ l_2 = 0.6$ and Fig. 5(b) with $l_1 = l_2 = 0.2$. At the same time, the utility loss is essentially not sensitive to α_2 and α_3, as seen

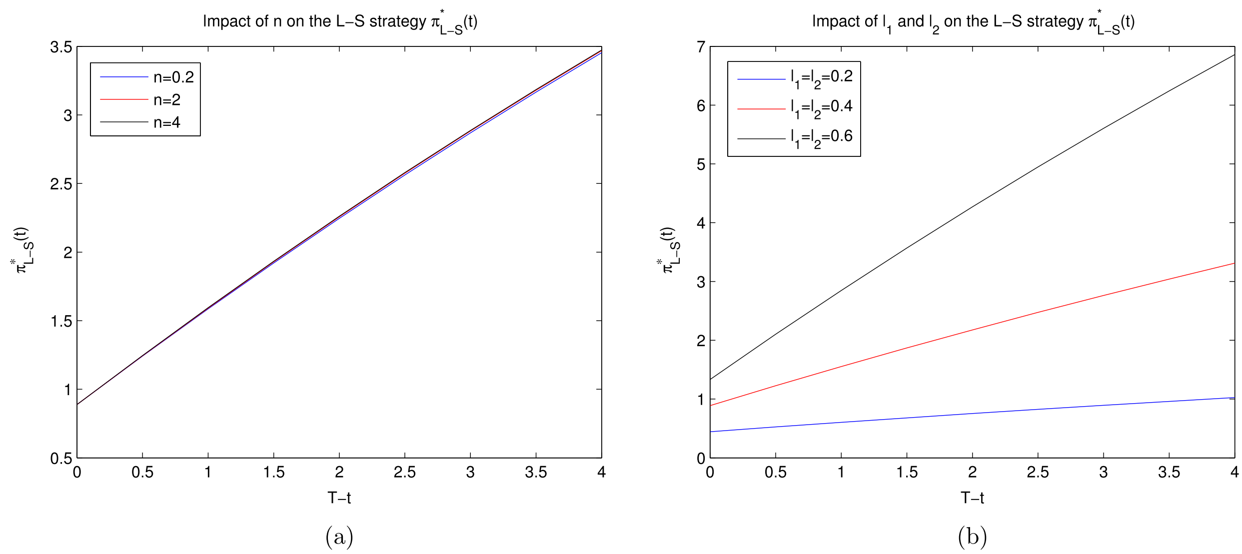

Fig. 1. Impact of n and $l_1 + l_2$ on the optimal investment strategy.

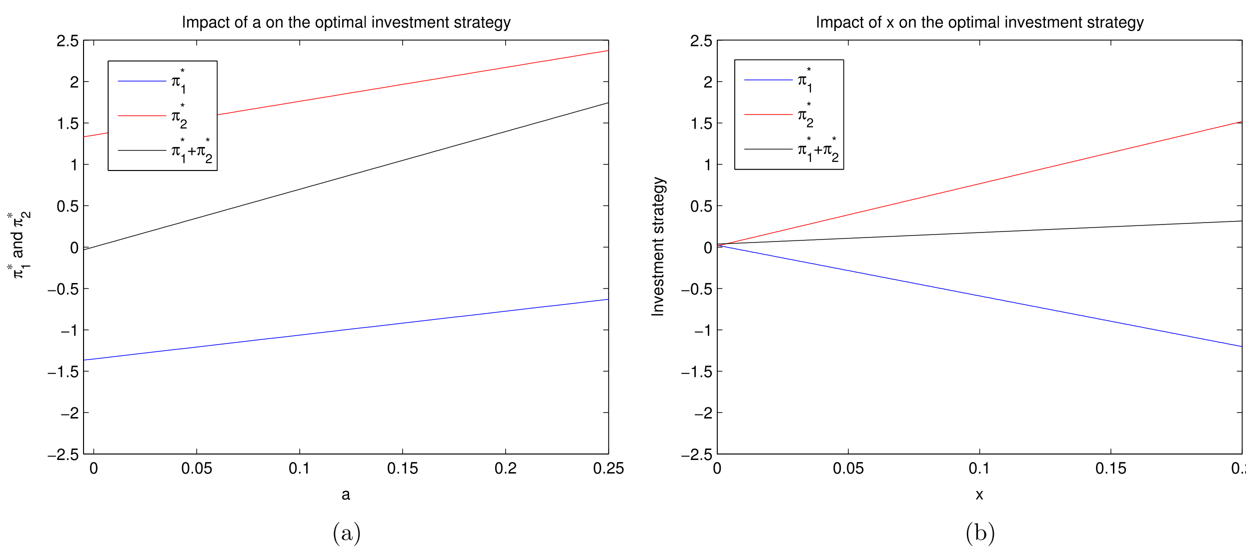

Fig. 2. Impact of a and of x on the optimal investment strategy with $t = 2$.

in Fig. 5(d–f). All this shows that ambiguity aversion to the financial market is the main cause of utility loss.

The most pronounced effect observed in Fig. 5 is the fact that utility loss increases dramatically with respect to the horizon time $T - t$. This means that for medium and long-term horizons, being ambiguity averse is potentially very expensive, but it is a more affordable attitude for short time horizons.

Next, we study the situation in which there exists mispricing in the market, but the AAI does not realize this point, so she think that the optimal investment strategy is $\pi^*|_{l_1=l_2=0} = \pi^*|_{x(t)\equiv 0}$, and gets the corresponding optimal value function V^{IM}. Ignoring the mispricing can cause missing some investment opportunities, thus this causes some loss in the utility. Correspondingly, we define the utility loss function

$$L^{IM} = 1 - \frac{V}{V^{IM}}.$$

It is shown in Fig. 6(a, b) that this utility loss will increase with the decrease of ambiguity aversion α_1. This can be explained by the decrease in hedging demand: the higher α_1 is, the less the fully cognizant insurer would have liked to exploit mispricing, the less ignoring those opportunities will cause loss of utility. We notice that when $l_1 \neq l_2$, the utility loss is more sensitive to α_1 and is larger than that in the case of $l_1 = l_2$. This is due to the fact that, when $l_1 = l_2$, utility loss is caused by the L-S strategy and π^*_{L-S} has a negative relationship with α_1.

It is shown in Fig. 6(c) that the smaller the mean reverting rate n is, the greater the utility loss function is. This is an indication that, because the mean reverting rates n measure the speed at which the risk premium $a(t)$ of the stocks gets back to the long-run mean m, when n is smaller, the investor will choose to adopt greater leverage on the mispricing of stocks since it persists for longer. We notice that the loss utility is not sensitive to the change of n and the effect of n on the loss utility will increase with the increase of liquidity. As a result, this leads to more utility loss when mispricing opportunities are ignored.

Similarly, Fig. 6(d) also informs us about what ignoring mispricing can bring in terms of utility loss when the economic conditions are conducive to more dynamic mispricing between stocks 1 and 2. Indeed, when the liquidities l_1 and l_2 are higher, one might in principle believe that mispricing opportunities decrease, but in reality, at a given mispricing level x, the effect on utility loss will

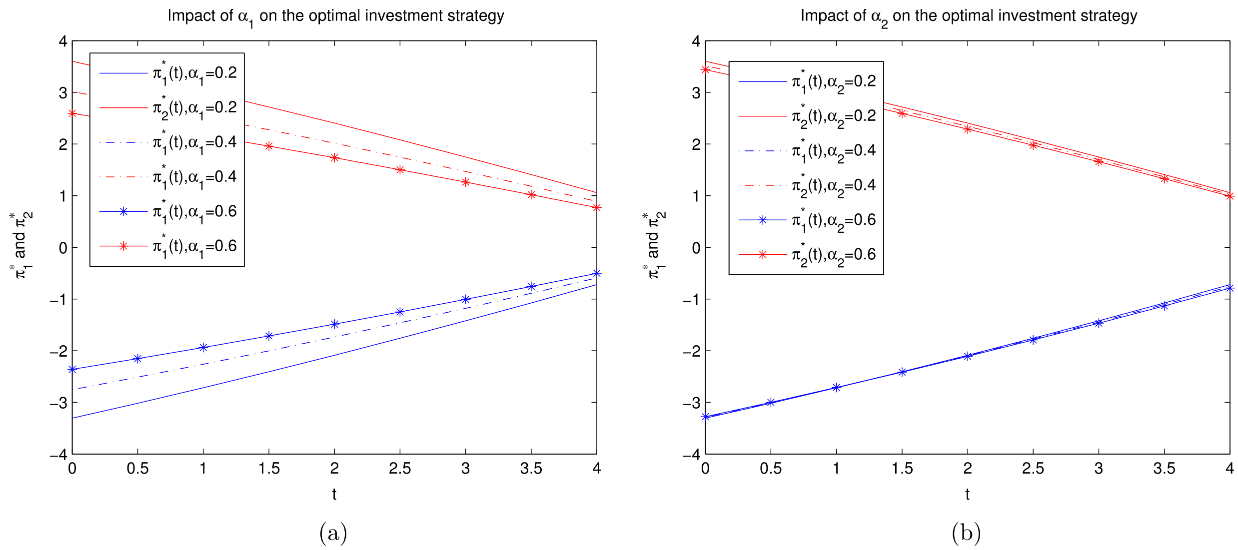

Fig. 3. Impact of α_1 and α_2 on the optimal investment strategy with fixed a and x.

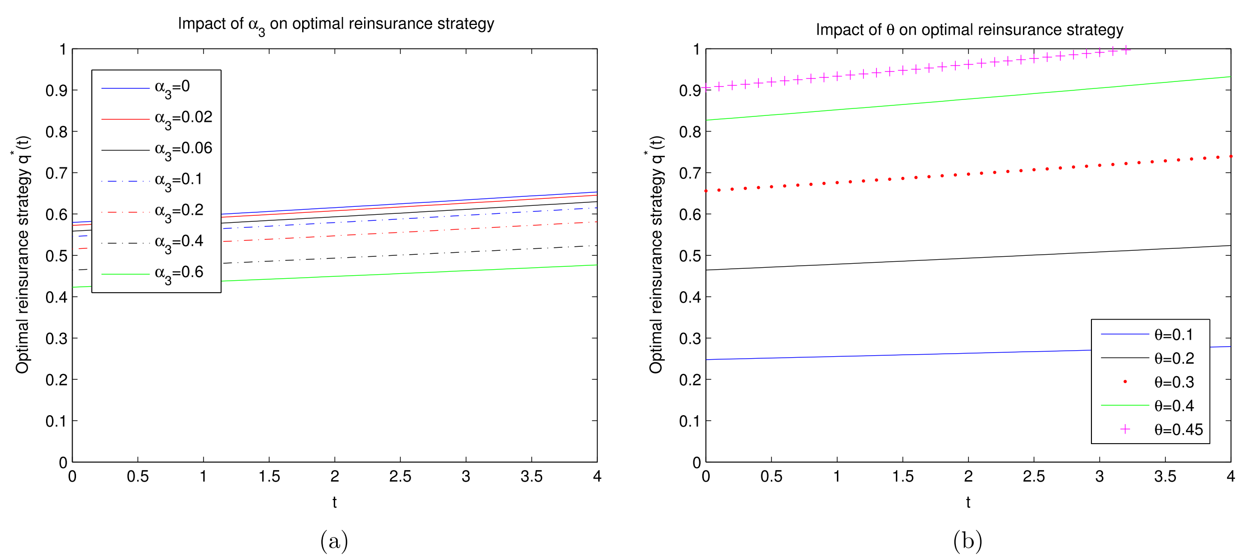

Fig. 4. Impact of α_3 and θ on the optimal reinsurance strategy.

be greater for higher liquidities, as seen in that figure. This must be interpreted by saying that for a given mispricing level, higher liquidities imply a higher reversion of the mispricing level to its mean 0, but also a faster subsequently reversion to similar levels as the original x, which the AAI who is cognizant of mispricing can take advantage of. The AAI who ignores this suffers the utility losses in Fig. 6(d). In addition, Fig. 6 hints at another piece of information: taking advantage of mispricing is more important for long-horizon investors than that for short-horizon investors. For example, in the case of $l_1 = l_2 = 0.4$, the effect is dramatic: when $T - t = 1$, the loss utility is less than 10%, but when $T - t = 4$, the loss utility approaches 90%.

Finally, we analyze utility loss from ignoring reinsurance. As we know, reinsurance is a main measure for the insurer to avoid her business risk, or transfer that risk. Moreover, reinsurance is a tool to help increase utility, as we now see. We define the utility loss

$$L^{NR} = 1 - \frac{V}{V^{NR}}.$$

Fig. 7 tells us ignoring the reinsurance will generate some utility loss, which connects the ambiguity aversion to the jump risk in the AAI's surplus, to the claim intensity, and to the reinsurance premium. Ambiguity aversion α_3 has a positive effect on the loss utility, i.e., the loss will increase with the increase of ambiguity aversion to jump intensity modeling. This is because the more the AAI is averse to errors in the claims model, the more the fully cognizant AAI will tend to depend on reinsurance, so the higher the loss will be if she ignores reinsurance as a risk-mitigation tool. Fig. 7(b) tells us that the intensity of claims arrival has an important role in the utility loss L^{NR}: intuitively, the demand for reinsurance should increase with an increase in claim intensity, and the figure shows that the corresponding loss from ignoring this risk-mitigation measure also increases in this case. On the other hand, before the AAI buys reinsurance, she will consider its price and should be more likely to buy more of it at a lower price. This should be captured in the safely loading factor θ. One expects that when θ is high, the AAI maybe choose to give up most reinsurance: indeed, we can see in Fig. 7(c) that when $\theta = 0.28$, the utility loss is less than 6% without reinsurance. Fig. 7 also shows that in all cases, longer time horizons have a marked effect on the utility loss from ignoring reinsurance, but such an insurer's loss would

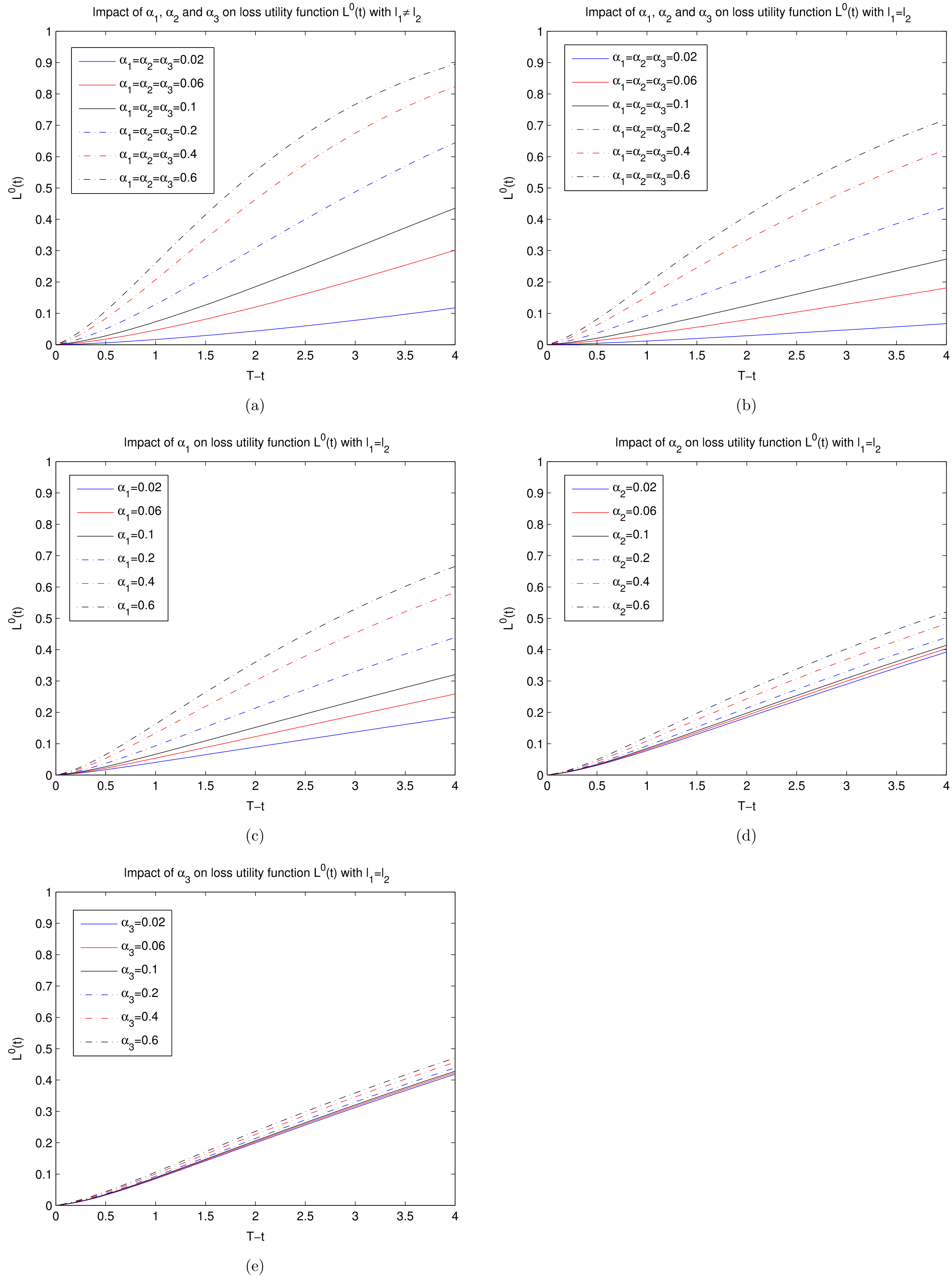

Fig. 5. Impact of α_1, α_2 and α_3 on the loss utility function $L^0(t)$.

be much smaller, and arguably negligible, for short time horizon, e.g. less than a year. Finally, we mention the effect of ambiguity on the insurance diffusion modeling risk: this ambiguity is captured by α_2, and we see that it does not affect the loss utility L^{NR} at all, see Fig. 7(d). The reason is plain to explain: L^{NR} only depends on the claim risk and is independent of the other risk.

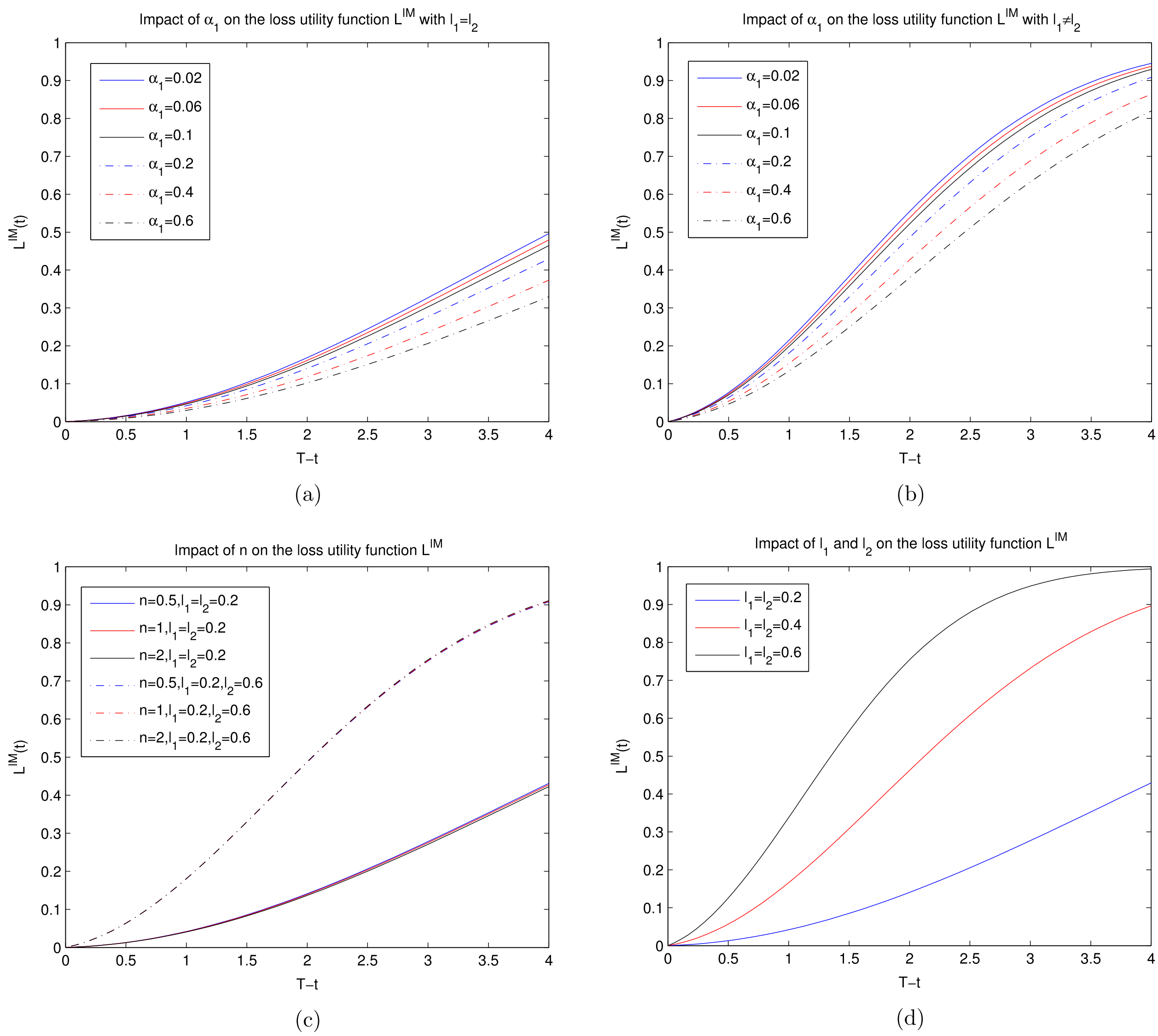

Fig. 6. Impact of α_1, n and $l_1 + l_2$ on the loss utility function $L^{IM}(t)$.

5. Conclusion

In this paper, we discuss optimal reinsurance-investment strategies for an ambiguity-averse insurer (AAI) with mean reversion and mispricing. The AAI's surplus is described by a jump–diffusion model and the insurer accepts ambiguity towards both the jump term and the diffusion term in the model, meaning that she is skeptical about her own ability to choose the corresponding model drift parameters, as she would using calibration or statistical tools based on insurance market data. We also consider a financial market consisting of one risk-free asset, one market index, and a pair of mispriced stocks which offer statistical (stochastic) arbitrage opportunities, which the AAI may take advantage of. The appreciation rate for the stock and the mispricing ratio are described by mean-reverting stochastic processes whose mean-reversion rates reflect liquidity constraints. The insurer is also suspicious of the financial model's veracity, calling its drift parameters into question as well. With these two sources of modeling ambiguity, she worries about the robustness of her results and decisions based on her estimated or calibrated model. Thus we formalize the insurer's ambiguity aversion to the insurance model and the financial market and use the so-called "homothetic robustness" modification of a classical exponential utility to study the problem of optimizing the AAI's utility given her ambiguity levels. These levels of modeling-risk aversion are described by one parameter for ambiguity in the diffusive term in the financial model, one parameter for ambiguity in the diffusive term in the insurance surplus model, and one parameter for ambiguity in the jump term in the insurance claims model. Using the dynamic programming approach, we derive the explicit optimal robust reinsurance strategy and the corresponding value function. Finally, we give numerical illustrations to analyze our results and make some practical recommendations. By studying our optimal investment strategy's sensitivity to various parameters, we uncover that liquidity has an important role in the so-called long–short (L–S) strategy, the one which takes advantage of statistical arbitrage afforded by mispricing. We also find that the positions in the two stocks decrease with respect to time-to-maturity and to ambiguity. We further find that the insurer tends to decrease her reinsurance level with the increase of her ambiguity aversion and as the reinsurance premium increases. In order to show the importance of ambiguity aversion, mispricing, and reinsurance, we also define and discuss loss utility functions from ignoring these effects. Ignoring ambiguity can bring higher profit if the model is correct; using ambiguity appears to be expensive for longer investment horizons. Ignoring mispricing

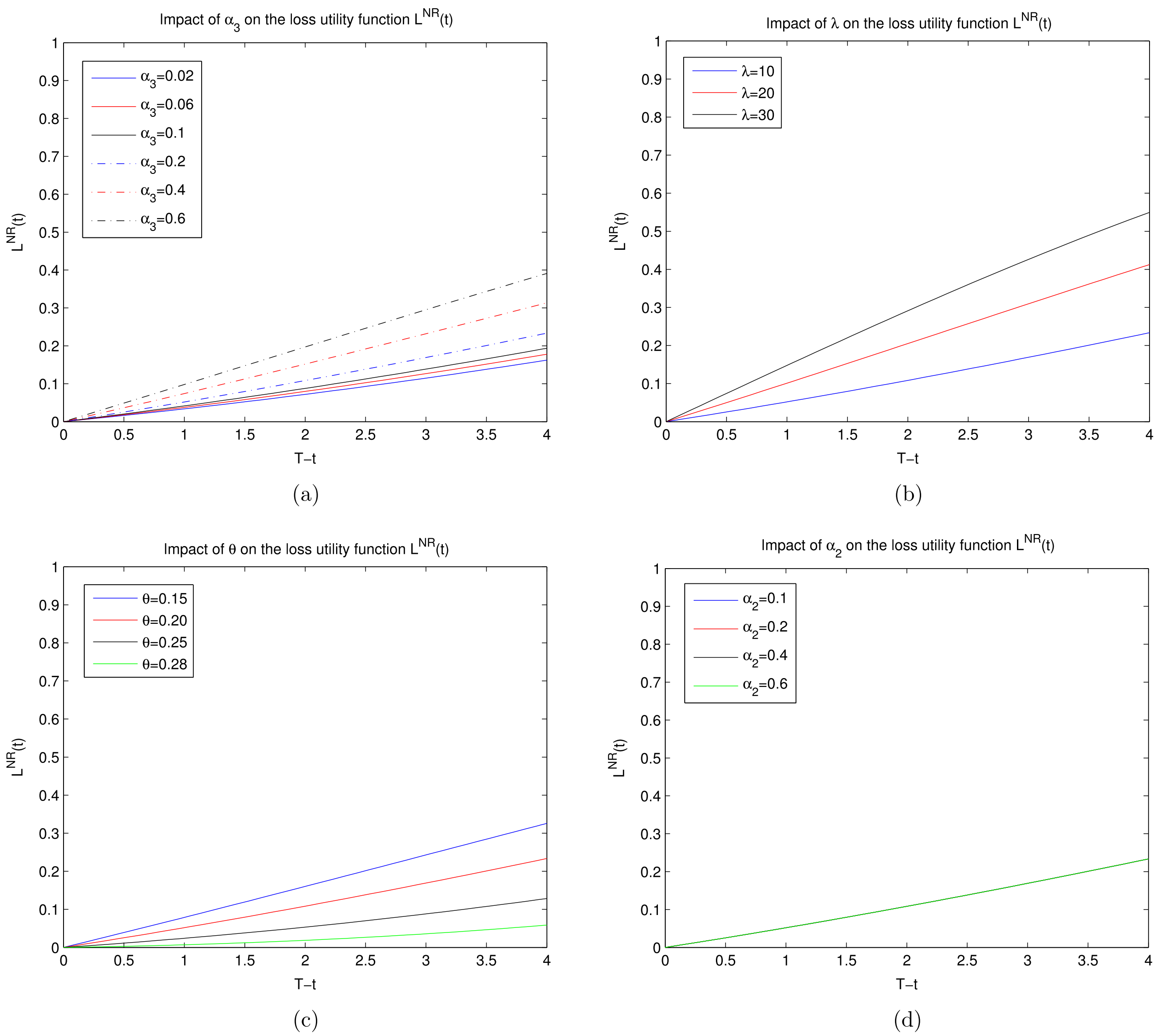

Fig. 7. Impact of α_3, λ, θ and α_2 on the loss utility function $L^{NR}(t)$.

and reinsurance results in losses of utility which are significant for medium and longer investment horizons.

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